PART I
Theory and Concepts

CHAPTER 1

TIDES AS WAVES

1.1 What is the tide?

Every reader of this book will have some notion of what is meant by the word "tide" as applied to the ocean. Some will think of the daily or twice-daily rise and fall of the water on the face of a cliff or around the pilings of a pier, others of the advance and retreat of the water over a shallow foreshore, and still others may think of the variable horizontal flow of the water that carries their ship off course, sometimes in one direction, sometimes in another. The tide is all of these things, but more generally we will define the ocean tide as the *response of the ocean to the periodic fluctuations in the tide-raising forces of the moon and the sun.* This response is in the form of long waves that are generated throughout the ocean. They propagate from place to place, are reflected, refracted, and dissipated just as are other long waves. Thus it is that the tide observed at a particular place is not locally generated, but is the sum of tide waves arriving from all over the ocean, each modified by its experiences along the way. To better understand the tide it will therefore be desirable to consider the characteristics of long waves as well as those of the tide-raising forces that produce them.

1.2. Waves

Wave motion in or along a medium is characterized by:

a) periodic vibration but no net transport of the particles in the medium,

b) propagation of energy along or through the medium, and

c) a restoring force that opposes the displacement of the particles of the medium.

When a sound wave travels through air, the particles experience a to-and-fro vibration, and the restoring force is provided by the pressure gradient. When a sound wave travels through a solid, the particles also experience a to-and-fro vibration, and the restoring force is provided by the elasticity of the material. When a wave travels along a taut string, the particles experience a transverse vibration, and the restoring force is provided by the tension in the string. When a wave travels along the surface of a body of water, the particles experience both a to-and-fro and an up-and-down vibration, and the restoring force is provided by a combination of gravity (acting through the hydrostatic pressure) and surface tension. Surface tension is the dominant restoring force only for ripples with 2 cm or less between crests, and these are called "capillary waves." For all longer water waves the dominant restoring force is gravity, and for this reason they are called "gravity waves." Surface chop, sea, swell, tsunamis, and tides are all gravity waves.

The terminology used to describe waves is illustrated in Fig. 1a and b. In Fig. 1a the wave form is viewed perpendicular to its direction of travel at an instant in time; Fig. 1b depicts the variation in water level at a fixed location over an interval of time as the wave passes. The *wavelength* (λ) is the distance between successive *crests* or successive *troughs*. The *range* (*R*) is the vertical distance of

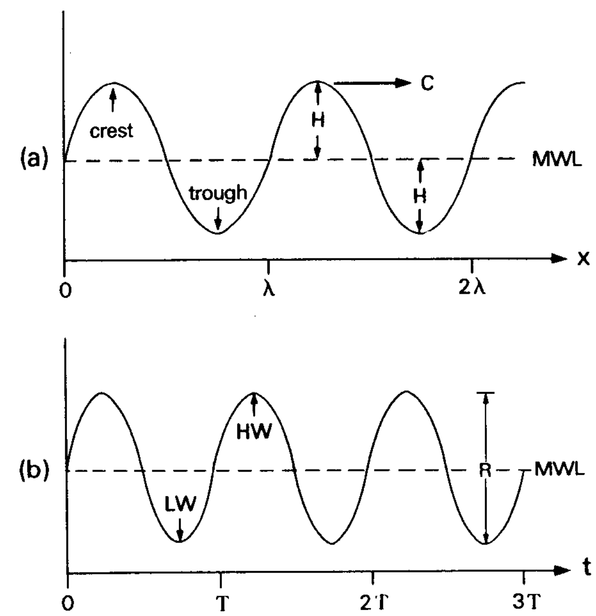


Fig. 1. Sinusoidal wave form as seen (a) in space at an instant of time and (b) at a fixed location over an interval of time.

the crest above the trough, or of the *high water* (HW) above the *low water* (LW). The terms crest and trough are more commonly used in connection with waves that are short enough to reveal their wave form to the eye. The terms HW and LW are more commonly used only in connection with the tide waves, which are much too long to reveal their form to the eye. The *amplitude* (H) is one half of the range. The *period* (T) is the interval between the passage of two successive crests, or between the occurrence of two HWs: successive troughs or any other identifiable parts of the wave form could equally well be used to define the period. The *frequency* (f) is the number of periods (or cycles) occurring per unit time; therefore $f = 1/T$. The *wave speed* (c) is the horizontal rate of advance of all parts of the wave form (crests, troughs, etc.). Since a travelling wave advances one wavelength in one period, $c = \lambda/T$.

A sinusoidal wave form, such as that in Fig. 1, can be generated as the product of the amplitude times the sine or cosine of a continuously increasing angle, called the *phase*. The angle by which the phase of a wave lags behind the phase of a reference wave is called the *phaselag*. In tidal work, the cosine form is most commonly used, so that in Fig. 1b the height above mean water level (MWL) would be expressed as

$$h(t) = H \cos\left(2\pi f t - \frac{\pi}{2}\right).$$

With respect to a wave with phase $2\pi ft$, $h(t)$ would be said to have a phaselag of $\pi/2$. The rate at which the phase increases is called the *angular speed* (ω), and $\omega = 2\pi f$ radians per unit time. In tidal literature the angular speed is usually quoted in degrees per hour and given the symbol "n." The *wave number* (k) is the rate at which the phase changes with distance, and $k = 2\pi/\lambda$ radians per unit distance.

1.3. Surface gravity waves

It would admittedly be a rare occasion on which the actual sea surface could be adequately represented by a simple sinusoidal wave as in Fig. 1. However, quite complicated sea states may be represented as a composite of many such component waves, each with its own amplitude, wavelength, and direction of propagation. A long swell running on an otherwise calm sea closely resembles a single such component wave. Because the tide can usually be adequately represented as the superposition of a manageable number of these component waves, we will restrict our investigation of surface gravity waves to those of sinusoidal form.

A wave that is moving across the surface as a train of parallel crests and troughs is called a *progressive wave* . If it is moving in the positive x-direction, the height at distance x and at time t is given by

$$(1.3.1)\quad h_1(x,t) = H \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = H \cos(\omega t - kx)$$

This expression may be verified by considering that for an observer travelling with the wave speed $c = \lambda/T$ the phase would remain constant, because the increase due to the increase in t is offset by the decrease due to the increase in x. If the wave train is moving in the negative x-direction, the height is

$$(1.3.2)\quad h_2(x,t) = H \cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) = H \cos(\omega t + kx)$$

The superposition of two progressive waves that have the same amplitude and frequency but are travelling in opposite directions produces what is called a *standing wave*. Adding equations (1.3.1) and (1.3.2) and invoking some trigonometric relations give the following expression for the standing wave form:

$$(1.3.3)\quad h_s(x,t) = h_1 + h_2 = \left(2H \cos 2\pi\frac{x}{\lambda}\right)\cdot\left(\cos 2\pi\frac{t}{T}\right)$$

Figure 2 illustrates the formation of a standing wave from two oppositely directed progressive waves. From equation 1.3.3 and Fig. 2 it is clear that the period and wavelength of the standing wave are the same as those of the component progressive waves, that the amplitude of the rise and fall of the surface varies from zero to $2H$ according to the value of $\cos kx$, and that the phase of the rise and fall is everywhere the same or opposite, according to the sign of $\cos kx$. The places in the standing wave form at which the amplitude is zero are called *nodes*, and those at which it is maximum are called *anti-nodes*. The space between nodes is called a *loop*. Within each loop the phase is the same, but is different by 180° from that in adjacent loops. A

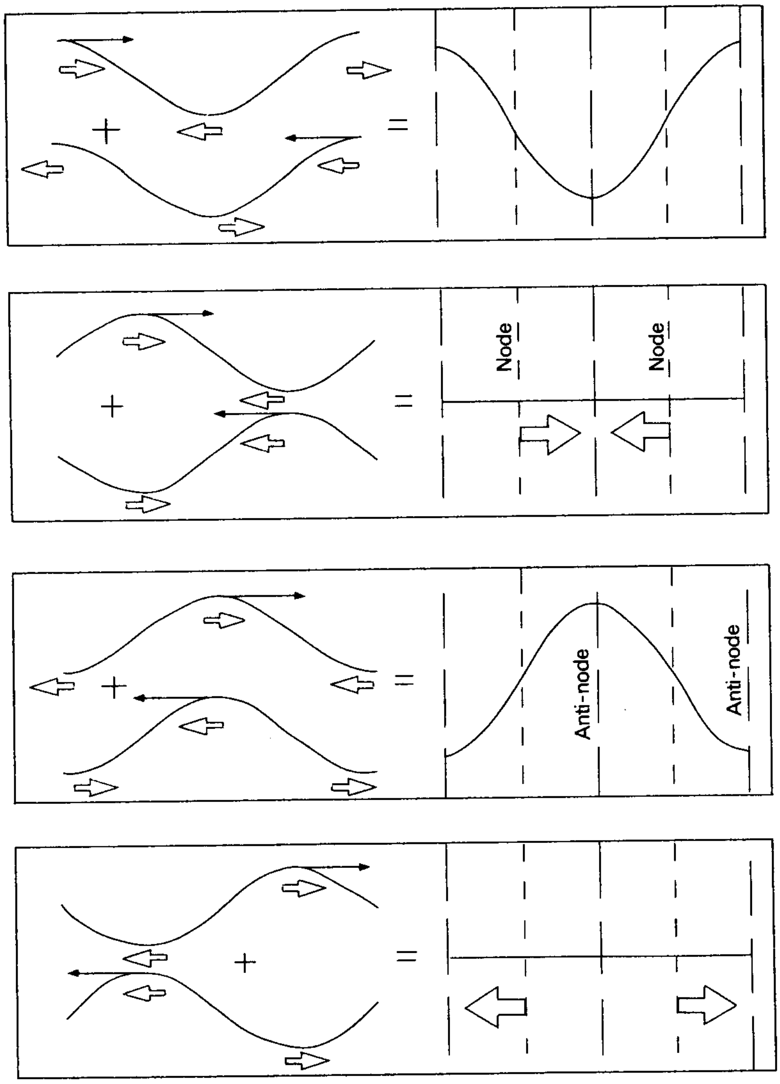


FIG. 2. Formation of a standing wave from two oppositely directed progressive waves. Open arrows show particle velocities; single line arrows show direction of wave propagation.

standing wave is frequently formed by the reflection of a progressive wave back upon itself, which is why the tide usually displays the character of a standing wave in coastal bays and inlets. In practice we will never encounter a pure progressive or standing wave; every wave will have some of the characteristics of each. The tide in the Strait of Belle Isle is an example of a regime that is neither purely standing nor progressive. The tide propagating out from the Gulf of St. Lawrence combines with the tide propagating in from the Atlantic, but, since the two do not have the same amplitude, only a partially standing wave is formed. At a true node there should be zero amplitude and a reversal of phase on either side: in the Strait of Belle Isle there is a degenerate node, exhibiting reduced amplitude and rapid spatial change in phase. When a standing wave is formed by reflection, the standing character is most nearly perfect near the reflecting barrier, because away from the barrier the incident wave has a larger amplitude than the reflected wave as a result of attenuation along their paths.

1.4 Long and short waves of small amplitude

In our theoretical consideration of waves we will implicitly assume that the amplitude is small with respect both to the wavelength and to the depth. The amplitude of a tide wave is always small with respect to its wavelength, but not always with respect to the depth, so we must expect some distortion of our results in shallow water. It can be shown that the wave speed of a sinusoidal wave in water of total depth D is

$$(1.4.1) \quad c = [(g/k) \tanh kD]^{1/2}$$

where g is the acceleration due to gravity, and that the horizontal particle motion (wave current) and the pressure associated with the passage of the wave both decrease exponentially with the depth by the factor $\exp(-kz)$, z being the depth from the surface. The magnitude of kD ($= 2\pi D/\lambda$) thus provides a criterion by which to categorize waves. *Short (or deep-water) waves* are those for which the wavelength is much less than the depth, and *long (shallow-water) waves* are those for which the wavelength is much greater than the depth. It must be remembered that the terms are relative, not absolute, and that a short wave may become a long wave on entering shallower water. For short waves kD is very large, and $\tanh kD$ is close to unity, so that the wave speed becomes $c_S = (g/k)^{1/2}$. For long waves the value of kD is very small, and $\tanh kD$ is approximately equal to kD, so that the wave speed becomes $c_L = (gD)^{1/2}$. Because for short waves the speed depends on the wavelength, they experience *dispersion*, the longer component waves travelling faster and becoming dispersed from the shorter component waves. This is why the long swells (forerunners) from a distant storm arrive first. Long waves do not experience dispersion, their wave speed depending only on the water depth. They do, however, experience *refraction* if one part of the wave front is travelling in shallower water than the others. The part of the wave front in the shallower water slows down, allowing the rest of the front to pivot around, changing the direction of propagation of the wave. As illustrated in Fig. 3, refraction is responsible for orienting waves parallel to beaches before they break on the shore. Short waves do not experience refraction; but, of course, they may become long waves on entering shallow water and then be refracted as in Fig. 3.

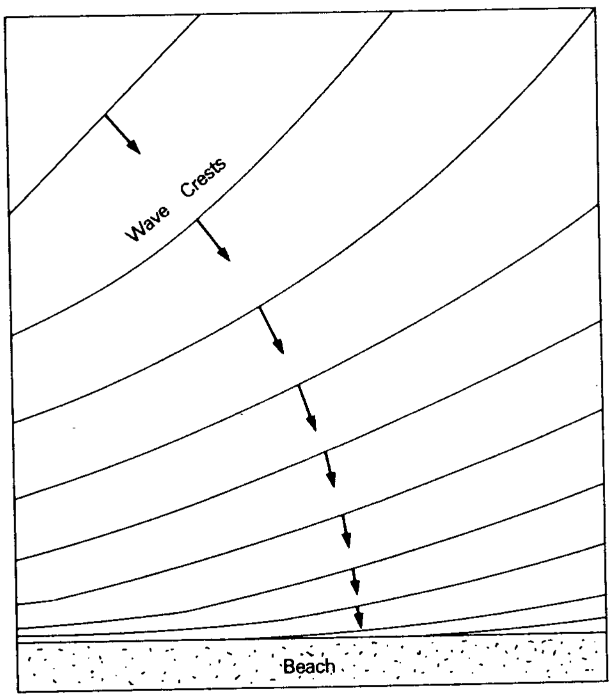


FIG. 3. Orientation of waves parallel to beach, by refraction.

The particle motion and the pressure fluctuation associated with the passage of a short wave decrease rapidly with depth, being only about 4% of their surface values at a depth of half a wave length. The particle motion and pressure fluctuation associated with the passage of a long wave are, however, virtually uniform over the depth (except for the frictional effect near the bottom). These are important facts to consider when planning subsurface pressure or current measurements. Since surface tides are long waves even in the deepest parts of the ocean, their signal may be detected by sensors at any depth, whereas the signal from short waves is effectively filtered out below a depth of a

TABLE 1. Properties of long and short waves.

	Short-waves (deep-water)	Long-waves (shallow-water)
Definition	$\lambda \ll D$	$\lambda \gg D$
Wave speed	$(g/k)^{1/2}$	$(gD)^{1/2}$
Particle motion	Decreases with depth.	Uniform with depth.
Wave pressure	Decreases with depth.	Uniform with depth.
Dispersion	Yes	No
Refraction	No	Yes

half a wavelength. The properties of long and short waves are summarized in Table 1. There are, of course, waves that are intermediate between the long and the short waves, and their wave speed is given by equation 1.4.1. However, since tides are always long waves, we shall confine our further considerations to long waves.

1.5. Particle motions in long waves

In this section we will develop expressions for both the wave speed and the particle speed in a long surface wave. This is being done partly to demonstrate the physical principles, and partly to emphasize the relation between these two speeds and between the particle motion and the wave form. We assume that the particle speed is uniform over depth and is a to-and-fro motion with the same period, but not necessarily the same phase, as the surface rise and fall. Consider a progressive wave moving from left to right in Fig. 4, with surface amplitude H. Let

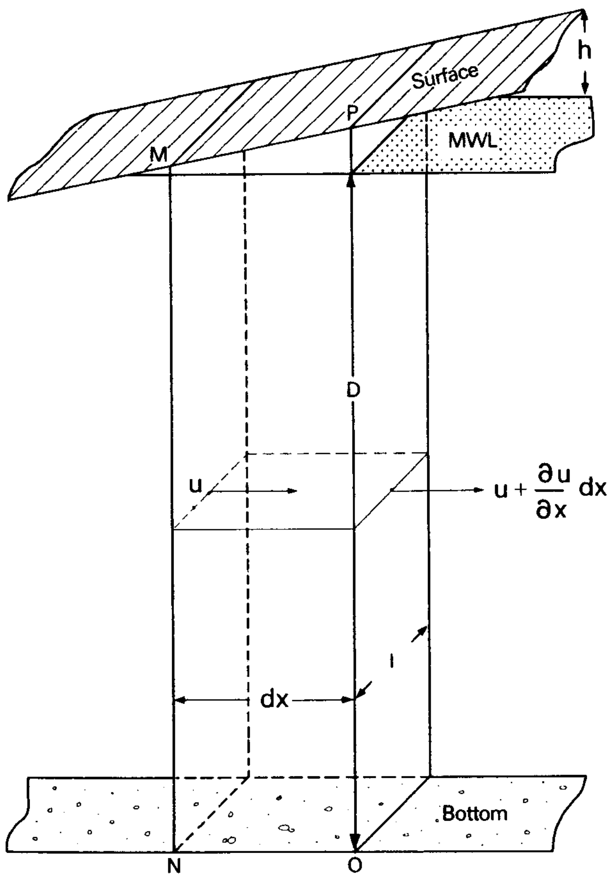


FIG. 4. Diagram to illustrate development of expression for wave speed and particle speed in long progressive surface wave.

the particle speed have amplitude U and phaselag θ with respect to the surface elevation. Therefore

$$(1.5.1) \quad h(x,t) = H \cos(\omega t - kx)$$
$$u(x,t) = U \cos(\omega t - kx - \theta)$$

In Fig. 4, MNOP is one side of a rectangular prism of unit thickness perpendicular to the page, with length dx, and height $D + h$. Its volume increases at the rate $(\partial h/\partial t)dx$. By the principle of continuity (conservation of matter) this must be equal to the rate at which water is entering minus the rate at which it is leaving through the sides of the prism. Neglecting the small height, h, with respect to the large depth, D, the rate at which water is accumulating inside the prism is

$$D[u(x,t)] - D[u(x,t) + (\frac{\partial u}{\partial x})dx] = -D(\frac{\partial u}{\partial x})dx.$$

Equating these two rates gives

$$(1.5.2.) \quad \partial h/\partial t = -D(\frac{\partial u}{\partial x})$$

Differentiating the expressions in 1.5.1 and substituting in 1.5.2 yields

$$-H\omega \sin(\omega t - kx) = -DkU \sin(\omega t - kx - \theta)$$

whence

$$(1.5.3) \quad \theta = 0, \text{ and } U = (\frac{H}{D})\cdot(\frac{\omega}{k}) = (\frac{H}{D})\cdot c.$$

Consider now a particle of water on the surface and assume that its acceleration equals the local acceleration, $\partial u/\partial t$ (a reasonable assumption for waves of small amplitude). The force per unit mass acting on the particle is the component of gravity parallel to the surface, $-g(\partial h/\partial x)$. By Newton's law of motion these two quantities must be equal, whence, upon differentiating the expressions in (1.5.1) and using the relations given in 1.5.3,

$$\frac{\partial u}{\partial t} = -g(\frac{\partial h}{\partial x})$$

or

$$-\omega(\frac{\omega}{k})(\frac{H}{D}) \sin(\omega t - kx)$$
$$= -gkH \sin(\omega t - kx)$$

so

$$(1.5.4) \quad (\frac{\omega}{k})^2 = gD = c^2$$

From 1.5.3, since $\theta = 0$, we have the very important result that in a progressive wave the particle motion is in phase with the surface wave form; so that the particle speed is greatest in the direction of the wave travel at the crest, greatest in the direction opposite to the wave travel at the trough, and zero midway between crest and trough. The particle speed beneath any point in the wave is in fact the wave speed multiplied by the ratio of the wave height to the depth. The wave speed is given by 1.5.4 as $c = (gD)^{1/2}$, as previously deduced from the more accurate expression in 1.4.1.

We will now examine the relation between particle motion and wave form in a standing wave. Just as we obtained the expression 1.3.3 for the form of a standing wave by adding the forms of two oppositely directed progressive waves, we may obtain the expression for the particle motion by adding the particle motions of two oppositely directed progressive waves. The particle speed in a wave travelling to the left is in phase with the wave form, and so must be given a negative sign. The addition gives the standing wave particle motion as

(1.5.5) $u_s(x,t) = U\cos(\omega t - kx) - U\cos(\omega t + kx)$,

or $u_s(x,t) = 2U \sin \omega t \cdot \sin kx$

From equation 1.3.3 we had the standing wave height as

(1.5.6) $h_s(x,t) = 2H \cos \omega t \cdot \cos kx$

Recalling that the sine of an angle is 90° out of phase with the cosine, we see from a comparison of 1.5.5 and 1.5.6 that in a standing wave the particle speed has maximum amplitude where the surface rise and fall has zero amplitude (i.e. at the nodes), and has zero amplitude at the anti-nodes. We also see that the particle speed achieves its local maximum everywhere when the wave form is flat, and is everywhere zero when the surface has its maximum distortion (i.e. at HW and LW). Figure 2 illustrates the relations between particle motion and wave form in progressive and standing waves.

To demonstrate that tide waves are indeed long waves ($\lambda \gg D$) and to emphasize the relation between particle speed (tidal stream) and wave speed, Table 2 lists the wavelength, wave speed and particle speed of a tide wave of one-metre amplitude and 12-h period travelling in various depths of water. Comparison of the values in the last two columns shows that the wave speed is everywhere much greater than the particle speed, but that while the wave speed decreases, the particle speed increases with decreasing depth of water. This is one reason that tidal streams are much more evident in coastal waters than in the open ocean.

1.6. Basin oscillations

Almost every physical system has a natural frequency at which it will oscillate when disturbed from its rest position or shape, until friction brings it once more to rest. The most obvious example is the pendulum (or the hair-spring or the quartz crystal) in a clock, whose natural period of oscillation is the time unit that is summed by the clock to record the passage of time. If a system is left undisturbed to oscillate at its natural frequency, it is said to be in free oscillation; if it is forced to oscillate at the frequency of an imposed force, it is said to be in forced oscillation. When the frequency of the driving force is equal to the natural frequency of the system, a large amplitude response may be obtained with the input of very little energy. This phenomenon is called *resonance*, and is explained by the fact that the driving force and the restoring force within the system act mostly in unison at forcing frequencies close to the natural frequency of the system, and mostly in opposition to each other at forcing frequencies far from the natural frequency. A simple example of a resonant system

TABLE 2. Characteristics of a tide wave of 12-hour period and 1 metre amplitude in various depths.

Depth (m)	Wavelength (km)	Wave speed (m/s)	Particle speed Amplitude (m/s)
5 000	9 600	220	0.04
500	3 000	70	0.14
50	960	22	0.44
5*	300*	7*	1.40*

*Since this depth is not very large w.r.t. the wave amplitude, the wave would be distorted, and these values inaccurate.

is a child seated on a swing that is being pushed by a friend; since the swing is pushed only at the end of each cycle, the frequency of the driving force is automatically matched to the natural frequency of the swing. The clock pendulum is another resonant system, operating on the same principle as the swing. A person singing in the shower may notice that a particular note causes a delightful reverberation; this is because the column of air in the shower stall is resonant at the frequency of that note.

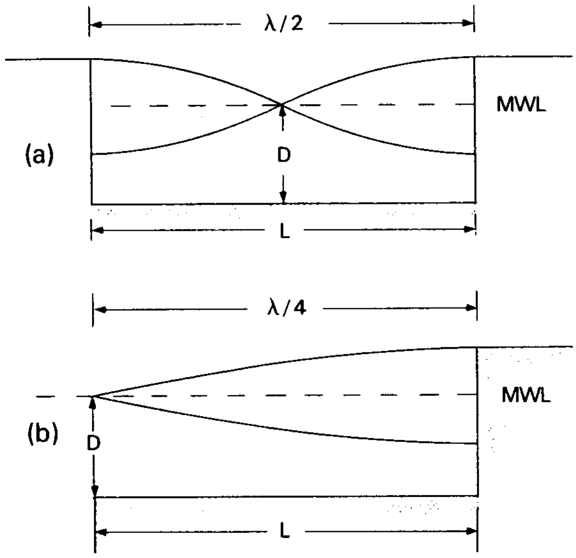


FIG. 5. Free oscillation, or *seiche*, in (a) a closed basin and (b) a basin open at one end.

The free oscillation of the water in a closed basin (bathtub, lake, etc.) takes the form of a standing wave with an anti-node at each end of the basin and one or more nodes between (Fig. 5a). If there is only one node, the length of the basin is half a wavelength, and the natural period of oscillation is given approximately as

$$(1.6.1)\quad T_n = \frac{2L}{c} = \frac{2L}{(gD)^{1/2}}$$

where L is the length of the basin and D is an average depth. Although it is less common, a closed basin could oscillate across its width as well as along its length. The free oscillation of the water in a basin open at one end (harbour, bay, inlet, etc.) takes the form of a standing wave with a node at the open end and an anti-node at the closed end (Fig. 5b). If there are no other nodes between the ends, the length of the open basin is a quarter wavelength, and the natural period of oscillation is given approximately as

$$(1.6.2)\quad T_n = \frac{4L}{c} = \frac{4L}{(gD)^{1/2}}$$

Free oscillations of water in basins (open or closed) are called *seiches*. Much of the early study of seiches was done on lakes in Switzerland, and the equations 1.6.1 and 1.6.2 are called Merian's formulae after one of the Swiss workers in this field.

Figure 6 illustrates how the tide at the entrance to an inlet off a large body of water drives a forced oscillation in the inlet in the form of a standing wave with an anti-node at the head of the inlet. If, as is usually the case, the inlet is shorter than a quarter of the tidal wavelength, the standing wave will have a virtual node outside the entrance. It is apparent that the amplitude of the tidal oscillation at the head of the inlet is greater than at the entrance, and that the amplification would be greatest if the node fell right at the entrance; the latter situation corresponds to the condition for resonance. If L is the length of the inlet, D its mean depth, $c = (gD)^{1/2}$ the wave speed in the inlet, and T the tidal period, then the tidal wavelength is cT and the portion of a wavelength within the inlet is L/cT. This represents a phase angle along the x-axis from the head of the inlet of $kx = 2\pi L/cT$. Thus, if H_2 is the amplitude at the head and H_1 that at the entrance of the inlet, by 1.5.6,

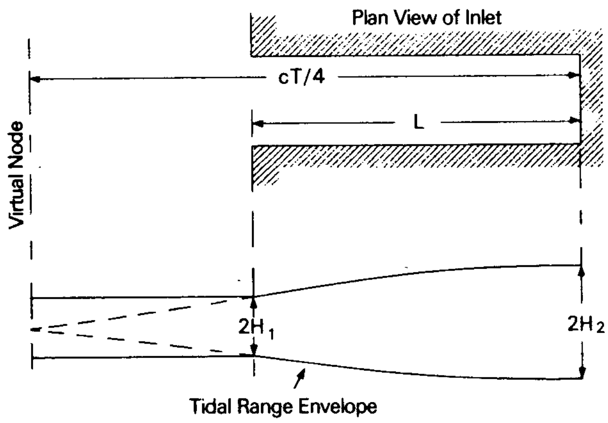


FIG. 6. Amplification of tide in an inlet, driven as a forced oscillation by the tide in a large body of water at entrance.

$$(1.6.3)\quad H_1 = H_2 \cos \frac{2\pi L}{cT}$$

$$\text{or}\quad \frac{H_2}{H_1} = \sec \frac{2\pi L}{cT}$$

The amplification factor, H_2/H_1, in 1.6.3 is seen to be infinite for $4L = cT$, which is the resonance

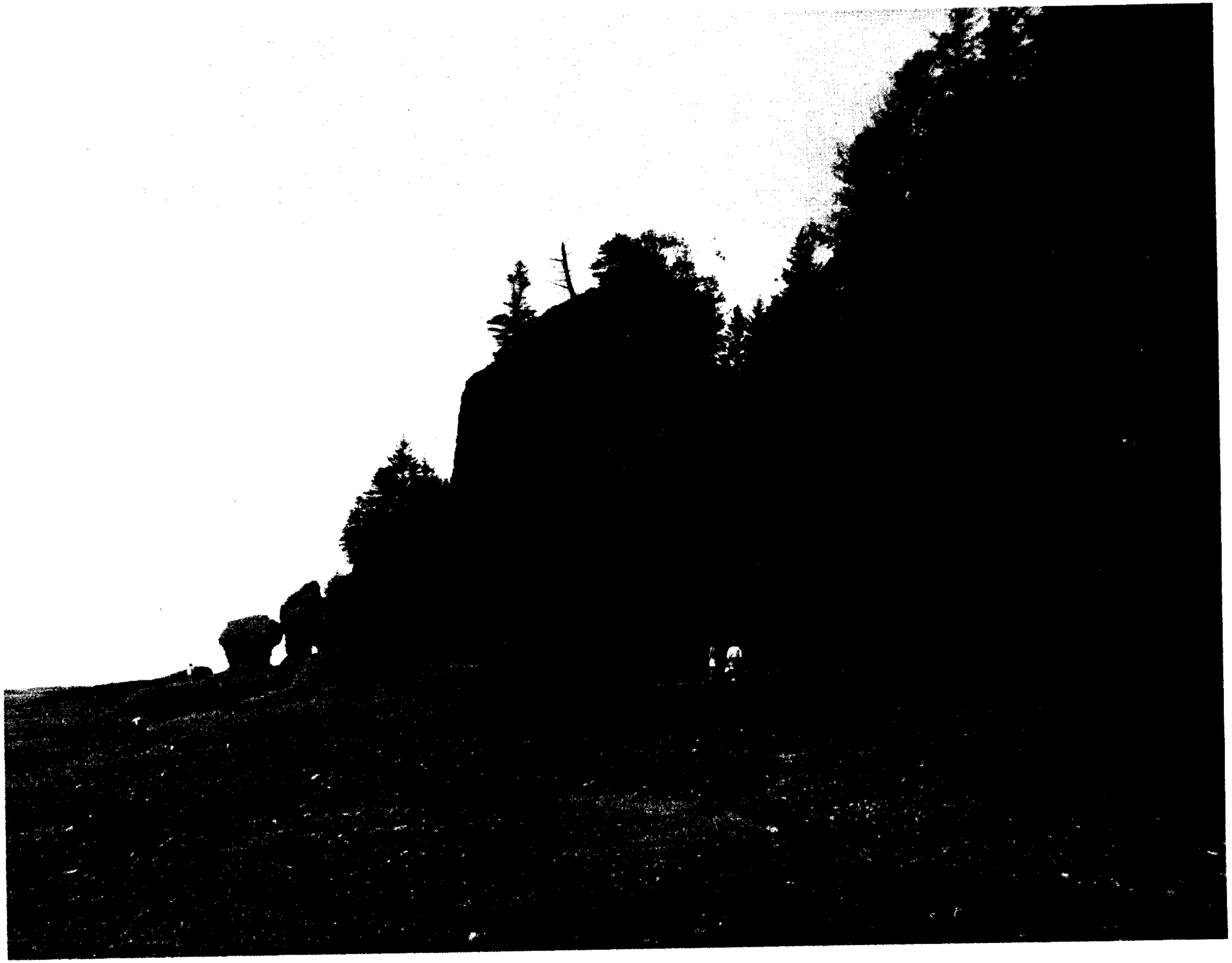


PLATE 1. Hopewell Rocks ("the flowerpots") at Cape Hopewell, New Brunswick, on Chignecto Bay at the inner end of the Bay of Fundy, at low water. The rocks have been eroded and formed into unusual shapes by water and sand suspended in the strong tidal stream. (Photo courtesy of the Canadian Government Office of Tourism.)

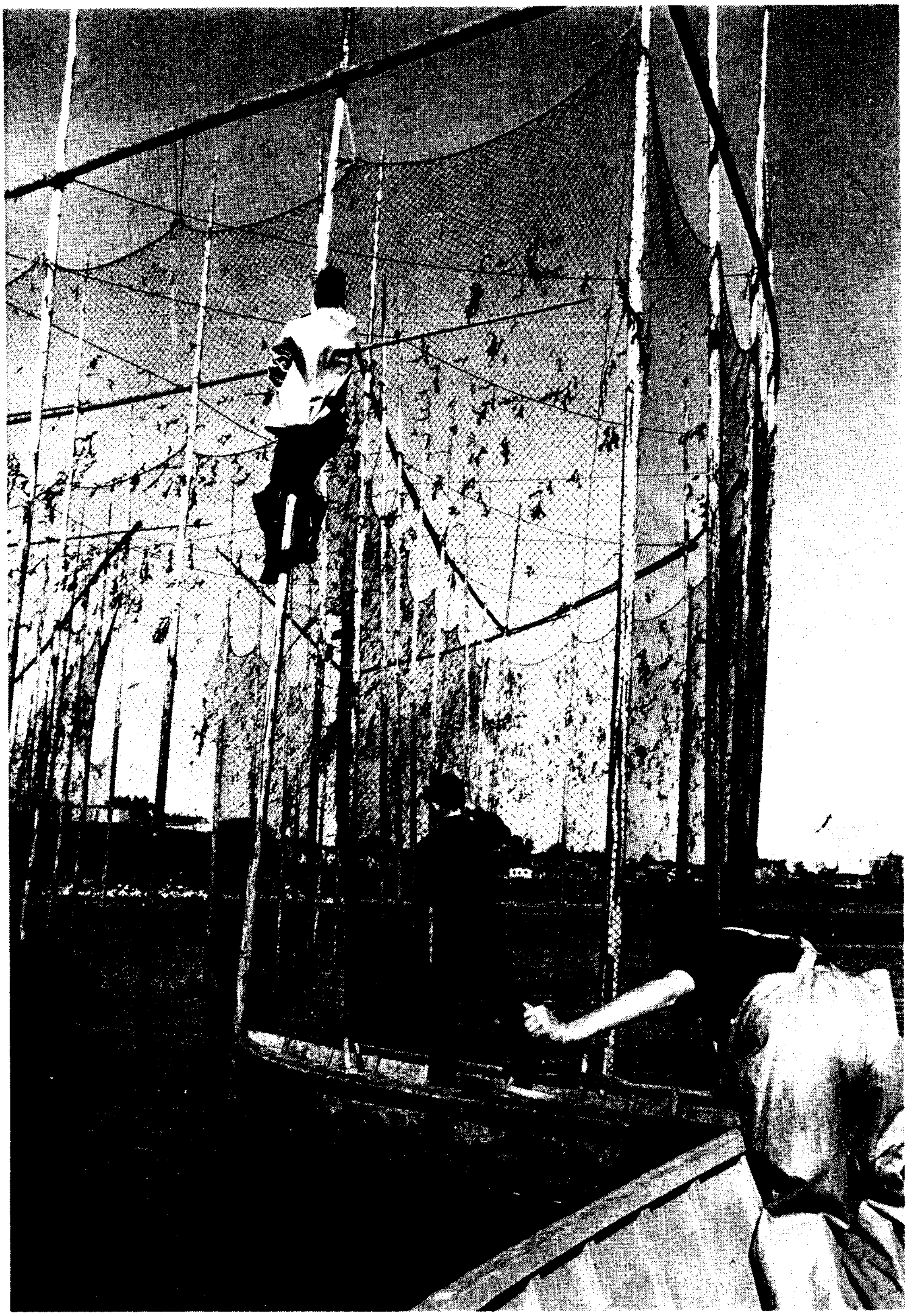


PLATE 2. Fishermen checking salmon fishing weir at low water near Saint John, New Brunswick. Fish are carried into and trapped by the weir because of the strong tidal flow: they are then fished out of the weir at low water. (Photo by R. Brooks, NFB Phototeque, 1964.)

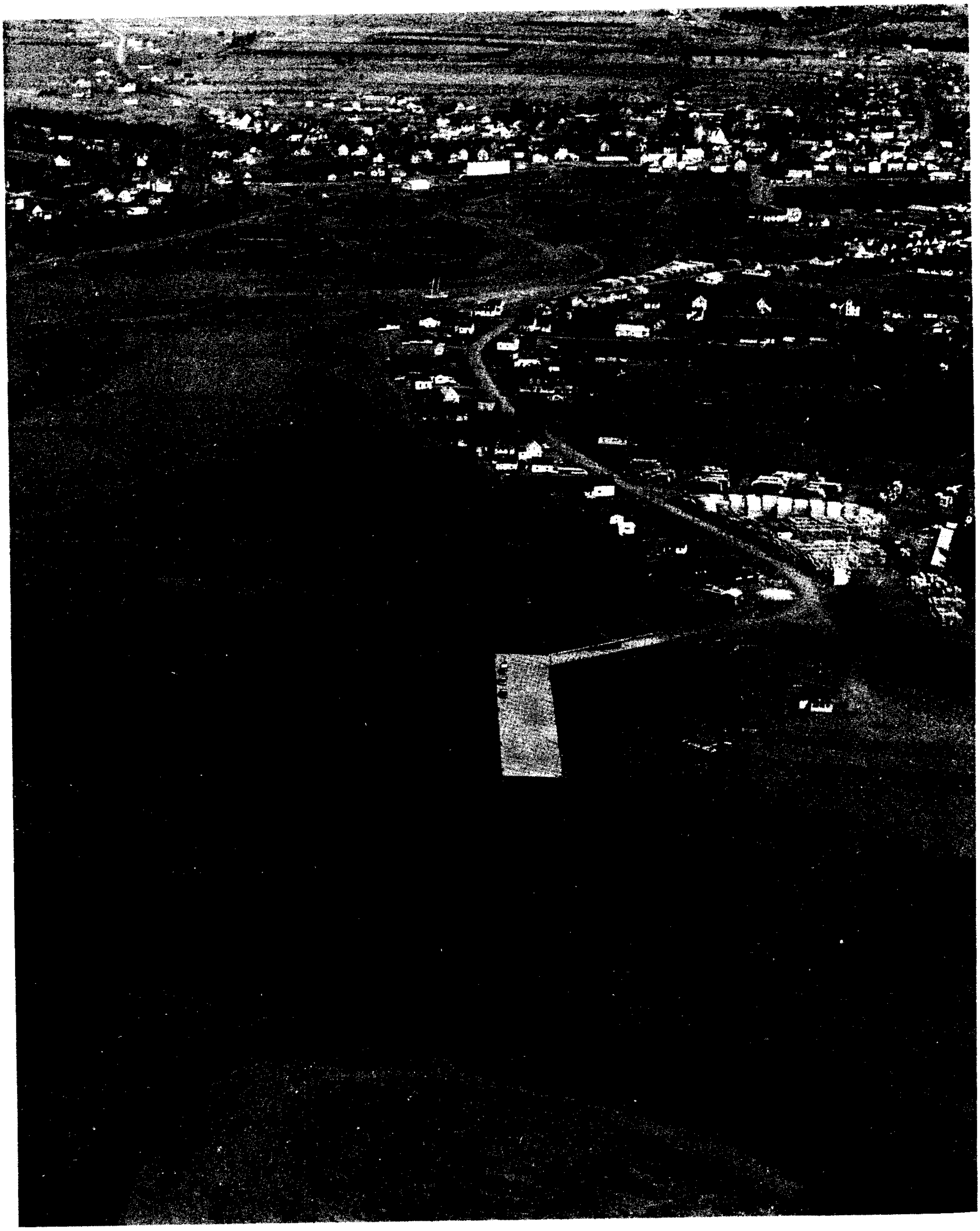


PLATE 3. View of jetty and "mattress" at low water, Parrsboro, Nova Scotia, on the north shore of Minas Basin, at the inner end of the Bay of Fundy. (Photo by R. Belanger, Bedford Institute of Oceanography.)

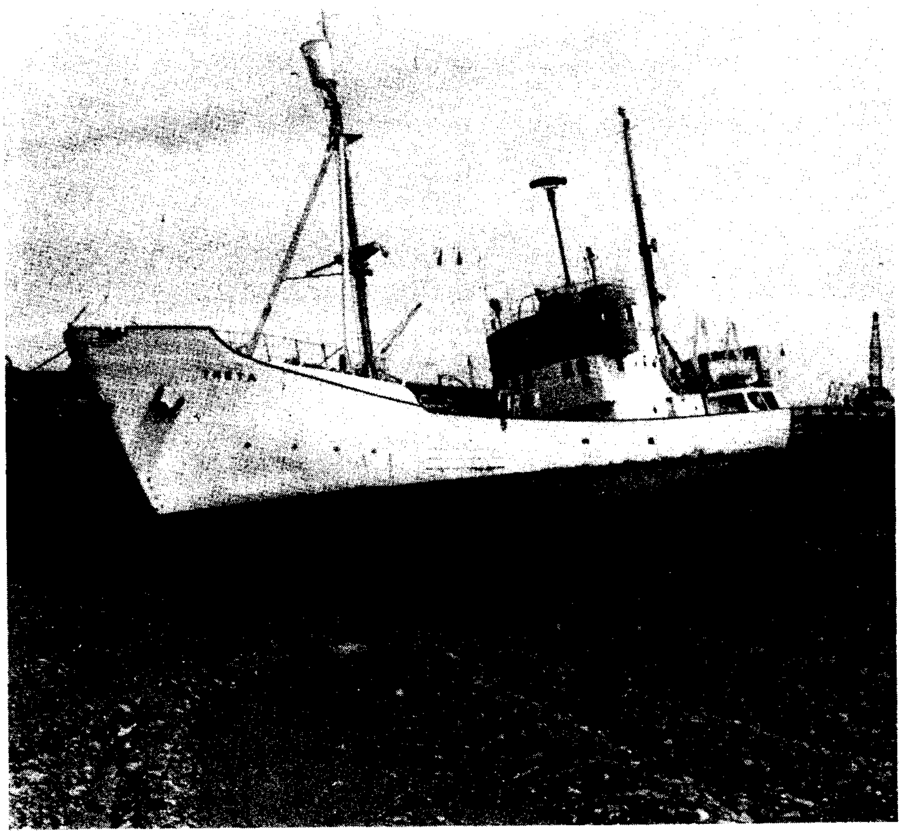
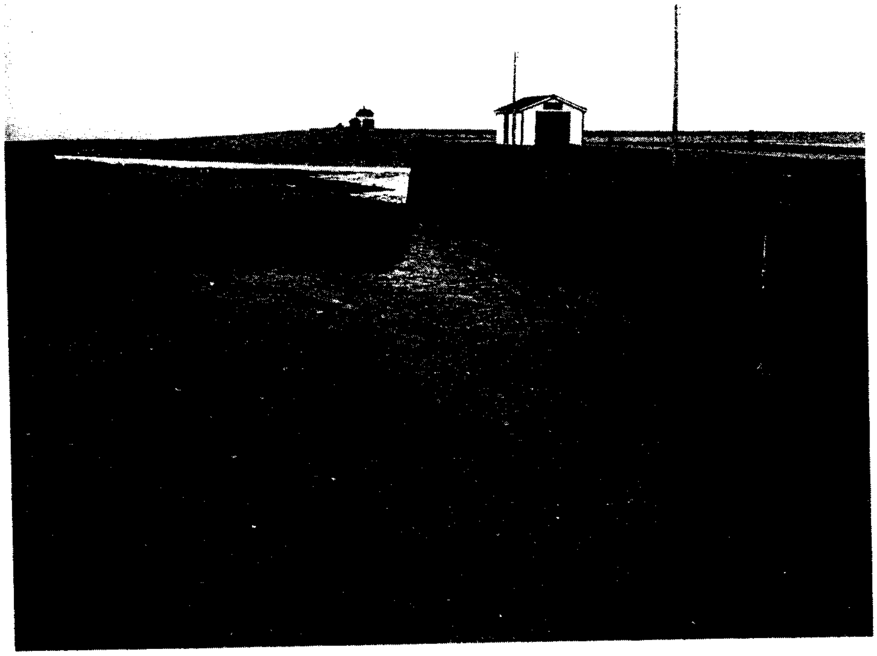


PLATE 4. MV *Theta* resting on wooden "mattress" beside jetty at low water, Parrsboro, Nova Scotia. (Photo by Canadian Hydrographic Service, 1960.)



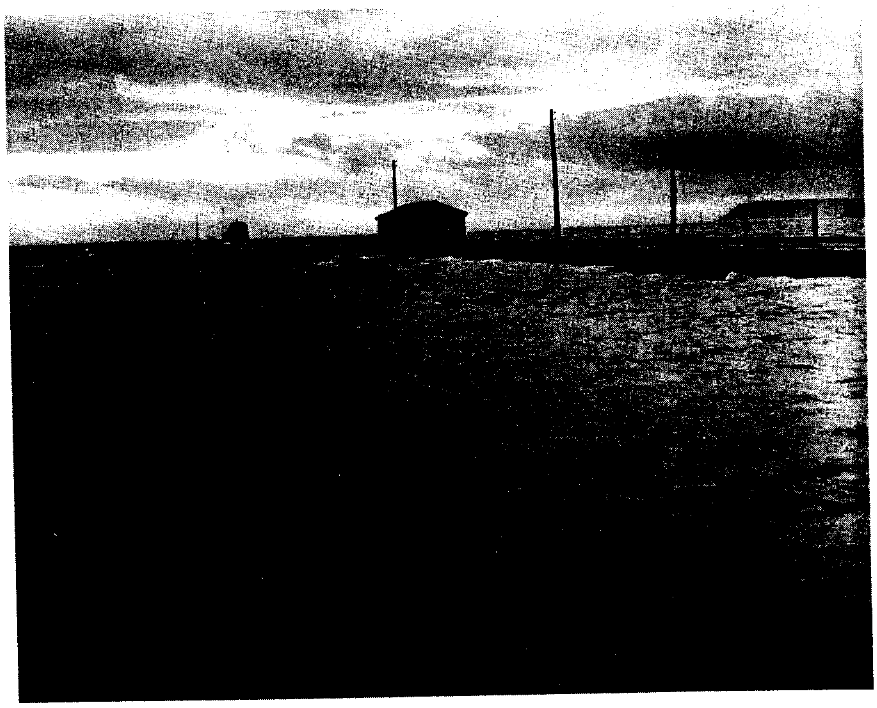


PLATE 5. Corresponding views of jetty at Parrsboro, Nova Scotia, at extreme low water (*upper*) and high water (*lower*). (Photos by C. Blouin, NFB Phototeque, 1949.)

condition (tidal period, T, equal natural period, $4L/c$). However, friction, which we have neglected, becomes very important near resonance, and formula 1.6.3 should not be used for systems near resonance. The Saguenay fjord provides an example of tidal amplification in a system that is not near resonance. The length from the entrance off the St. Lawrence Estuary at Tadoussac to the head of the fjord at Port Alfred is 95 km (L), the mean value of the long-wave speed in the fjord is 40 m/s (c), and the tidal period is 12.4 h (T). From this, 1.6.3 gives the amplification factor as

$$H_2/H_1 = \sec(0.33 \text{ rad.}) = \sec 19° = 1.06.$$

The actual amplification of the tide range at Port Alfred over that at Tadoussac is 1.16. The extra amplification over that predicted is probably caused by shoaling (see section 1.12) of the tide wave in the shallow water near the head. An example of a system that is nearly in resonance with the semidiurnal (T = ½-d) tide is the system comprising the Gulf of Maine and the Bay of Fundy.

The ocean basins themselves have natural periods of oscillation, but their modes of oscillation are much too complicated to be revealed by the simple considerations above. However, calculation of the natural period of east–west oscillation of the Atlantic and Pacific oceans from Merian's formula, 1.6.1, provides the interesting information that the Atlantic Ocean is more closely tuned to the semidiurnal and the Pacific Ocean more closely tuned to the diurnal tidal frequencies. Taking east–west widths of 4 500 and 8 000 km, respectively, for the Atlantic and Pacific and a mean depth of 4 000 m for both, 1.6.1 gives 12.6 and 22.3 h, respectively, as the natural periods of the Atlantic and Pacific for east–west oscillation. Pacific tides are indeed observed to have much more diurnal character in general than the Atlantic tides.

1.7. Internal waves

These are waves that occur below the surface at the interfaces between layers of water of different densities (i.e. in "stratified" water). They may exist independently from any surface wave, but are sometimes induced as a secondary effect of surface waves. Internal tides are frequently formed by the partial reflection of a surface tide wave at a sudden rise in bottom topography. The simplest case to consider is that of a wave at the interface in a

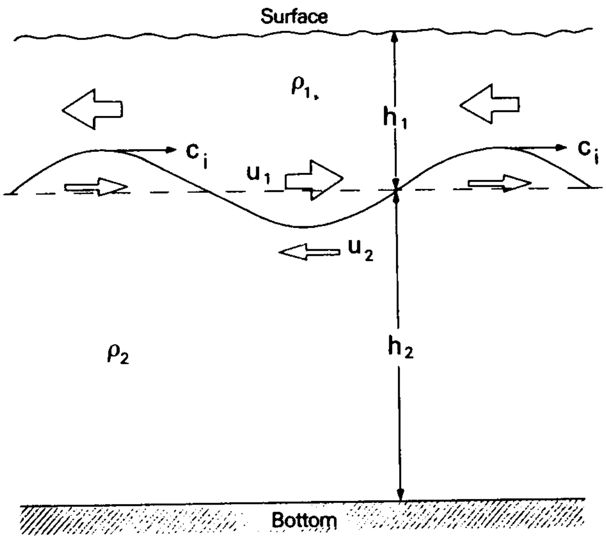


FIG. 7. Internal wave at the interface of a two-layer system. Hollow arrows show particle velocities; single line arrows show direction of wave propagation.

two-layer system as shown in Fig. 7. The subscripts, 1 and 2, refer to the upper and lower layers, respectively, and ρ is the density, h the layer thickness, and u the particle velocity (with amplitude U). The wave form at the interface is that of a long progressive wave traveling from left to right with wave speed c_i. The restoring force in this internal wave is not the full force of gravity, g, per unit mass, but is the buoyancy force $g\Delta\rho/\rho$, where $\Delta\rho$ is the density difference $\rho_2 - \rho_1$, and ρ is the mean density.

By reasoning that is just a little more difficult than that in section 1.5 for a surface wave, it can be shown that

$$(1.7.1) \quad c_i^2 = \frac{g(\rho_2 - \rho_1)}{\left[\frac{\rho_1}{h_1} + \frac{\rho_2}{h_2}\right]}$$

$$(1.7.2) \quad -U_1 = \frac{c_i H_i}{h_1}$$

and

$$(1.7.3) \quad U_2 = \frac{c_i H_i}{h_2}$$

where H_i is the amplitude of the internal wave at the interface. If we had wished, we could have treated surface waves as special cases of internal waves at the air–water interface, taking ρ_1 and ρ_2 as the densities of air and water, h_1 as the thickness of the atmosphere, and h_2 as the depth of the water. Putting $\rho_1 \ll \rho_2$ and $h_2 \ll h_1$ to comply with this, reduces 1.7.1 approximately to

$$c_i^2 = \frac{g(\rho_2)}{\left[\frac{\rho_2}{h_2}\right]} = gh_2,$$

in agreement with our previous expression 1.5.4. If, as is always the case for stratified water, ρ_2 and ρ_1 are nearly equal, 1.7.1 simplifies to

$$(1.7.4) \quad c_i^2 = g\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{h_1 h_2}{h_1 + h_2}\right)$$

and if it is further assumed that the upper layer is much thinner than the lower layer, as is frequently the case, this simplifies further to

$$(1.7.5) \quad c_i^2 = gh_1\left(\frac{\Delta\rho}{\rho}\right)$$

Admittedly the two-layer system of Fig. 7, with its discontinuity in density and particle velocity at the interface, could never occur in a natural body of water. However, the equations 1.7.1, 2 and 3 reveal the following important characteristics of internal waves:

1) Their wave speeds, and hence their wavelengths, are much less than those of surface waves of the same frequency.
2) The particle velocities of internal waves, unlike those of surface waves, may reverse phase and have different amplitudes at different depths.
3) They can exist only in stratified water.
4) They may have very large amplitudes (tens of metres) because the restoring force is so small.
5) Although their vertical amplitude is zero at the free surface, their particle velocities are usually greatest there, because of a thin surface layer of less-dense water.

Internal tides are internal waves of tidal frequency, and these have been observed in the St. Lawrence Estuary. Semidiurnal internal tides were observed in the estuary below Tadoussac with wavelengths of about 60 km. Their presence helped to explain the tidal streams in the area, which could not be satisfactorily accounted for by the surface tide alone. The water column in the estuary can be very crudely represented as two layers with $h_1 = 50$ m, $h_2 = 250$ m, and $\Delta\rho/\rho = 0.003$. Substitution of these values, along with $g = 9.8$ m/s^2 into 1.7.4 gives a wave speed of 1.1 m/s, or 4.0 km/h, corresponding to a wavelength of 49 km for the semidiurnal tidal period of 12.4 h.

1.8. Coriolis acceleration

Newton's classical laws of motion apply only when all measurements are made with respect to an inertial coordinate system, that is, one that is neither accelerating nor rotating. Thus, when measurements are made relative to a coordinate system fixed in the earth, allowance must be made for the rotation of the earth about its axis. This is done by providing two "fictitious forces," the *centrifugal force* and the *Coriolis force*, in addition to the apparent forces that cause acceleration of a body relative to the surface of the earth. A mass resting on the earth's surface is actually revolving about the earth's axis on a latitude circle once each day, and so is accelerating toward the centre of that circle. The inertia of the mass resists this centripetal acceleration, and, to an earth-bound observer, the mass appears to be pulled away from the axis by what he calls the centrifugal force. Since it varies

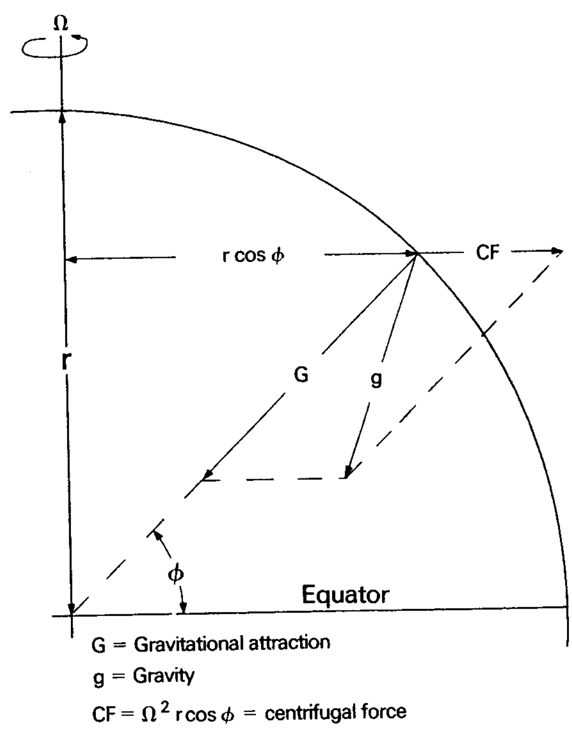


FIG. 8. Vector summation of earth's gravitational attraction (G) and centrifugal force due to earth's rotation (CF) to produce "gravity" (g).

only with latitude and not with time, the centrifugal force (CF) is conveniently combined with the earth's gravitational attraction (G) in what we know as "gravity" (g). Figure 8 depicts the vector addition of the two forces to give gravity, with the relative size of the centrifugal force vector greatly exaggerated for clarity. The centrifugal force is obviously greatest at the equator and zero at the poles, contributing to the fact that gravity is less at the equator than at the poles.

A body in motion relative to the surface of the earth experiences an acceleration to the right of its horizontal direction of travel in the Northern Hemisphere (to the left in the Southern Hemisphere), an acceleration that is proportional to its velocity and to the sine of the latitude. This acceleration is also a result of the earth's rotation, and is allowed for in the Coriolis force. Figure 9 attempts to illustrate the origin of this force. There is a Coriolis force on objects moving vertically and a vertical component of Coriolis force on objects moving horizontally, but we will consider only the horizontal component of the Coriolis force on objects moving horizontally. Imagine the earth to be covered with a frictionless film, the surface of which conforms to that of a level surface, i.e. is everywhere normal to the direction of gravity. N and S are the north and south poles, and Ω is the earth's angular velocity. As a body moves to higher latitude, the easterly velocity of the earth's surface decreases, and so the easterly velocity of the body relative to the earth increases. This is seen as an acceleration to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. Special cases of this are shown for a body projected south from N and for one projected north from S. Viewed from an inertial coordinate system both of these bodies would travel along the great circle NS, with no east–west velocity. Relative to the earth, however, they appear to follow the paths NM and ST, acquiring westerly velocity components as they move to lower latitudes, and so accelerating to the right in the Northern Hemisphere, and to the left in the Southern Hemisphere.

The Coriolis force due to the east–west velocity component arises from the fact that an easterly moving body experiences centrifugal force in excess of that included in gravity, and this excess centrifugal force has a component that accelerates the body along the level surface toward the equator. Similarly, a westerly moving body experiences a centrifugal force less than that in gravity and is accelerated along the level surface toward the pole. These accelerations are again seen to be to the right of the velocity in the Northern Hemisphere and to

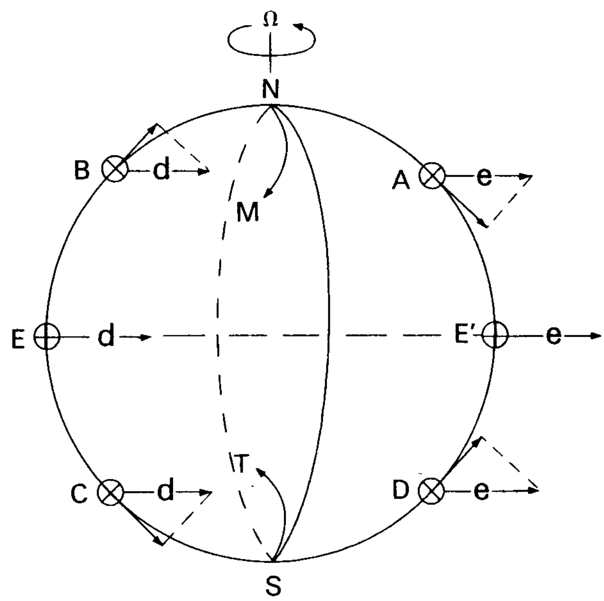


FIG. 9. Diagram to illustrate the origin of the Coriolis force, or acceleration.

the left in the Southern Hemisphere. This effect is illustrated at points A and D for easterly velocity and at points B and C for westerly velocity. The velocity vector at each point is directed into the page. The vectors **d** represent a deficit and vectors **e** an excess of centrifugal force over that allowed for in gravity for a body at rest on the surface. Their horizontal components are the horizontal Coriolis forces. Points E and E' show that for an east–west velocity at the equator the Coriolis force has only a vertical component. There is no horizontal Coriolis force for a north–south velocity at the equator because the rate of change of the earth's surface velocity with latitude is zero there. This can all be summed up in the statement that the horizontal component of the Coriolis force acting on a body moving with velocity v over the earth's surface acts to the right of the velocity in the Northern Hemisphere and to the left in the Southern Hemisphere, and has magnitude $2\Omega v \sin\varphi$, where φ is the latitude. $2\Omega \sin\varphi$ is called the *Coriolis parameter*, usually denoted as f.

The Coriolis force is rarely noticeable in laboratory-scale measurements, but is very significant in large-scale geophysical motions such as winds, ocean currents, and tides. It is this force that imparts the cyclonic and anti-cyclonic circulation to the atmosphere around low and high pressure regions and turns the ocean current systems into large circular gyres. It also acts on the tidal streams, changing the direction of propagation and the shape of the tide waves. When the tide propagates as a progressive wave along a channel in the Northern Hemisphere (NH), the range of the tide is observed to be greater on the shore to the right of the direction of propagation. This is because the tidal streams at HW are in the direction of propagation, and the Coriolis force acting on them moves water to the right until a slope of the surface is created to balance it. This raises the HW on the right shore and lowers it on the left. At LW the tidal streams are in the opposite direction, and the surface slope created to balance the Coriolis force lowers the LW on the right shore and raises it on the left. When the channel width is small compared to the tidal wavelength, only insignificant cross-channel tidal streams are required to create the surface slopes referred to above. Most channels are much narrower than the half wavelength of the surface tide required for resonance, but many may have a width comparable to the half wavelength of an internal tide. This is the case in the St. Lawrence Estuary, where the Coriolis force acts on an internal tide propagating seaward to produce strong cross-channel tilting of the interface between density layers, with correspondingly strong cross-channel tidal streams oppositely directed in the two layers.

1.9. Inertial currents

The Foucault pendulum is one of the few laboratory experiments that can demonstrate the effect of earth rotation (Coriolis force) on a body in motion. It consists of a heavy mass suspended on a long single filament swinging freely through a small arc. The vertical plane of the oscillation is observed to rotate through 360° in a period of (24/sinφ) hours, which period is referred to as the *pendulum day*. It is easiest to visualize this phenomenon for the special case of a pendulum suspended directly over the North or South Pole and swinging back and forth in a plane fixed in space, while the earth rotates once in 24 h beneath it. If it were possible to design such a pendulum to have a period of oscillation equal to one pendulum day, it would be observed to travel around in a circle once each half pendulum day. Again it is easiest to visualize this at one of the earth's poles: the pendulum would start tracing a circle at the centre of its swing and complete the circle when it returned again to the centre a half period later; by this time the earth would have rotated 180°, so the circle traced by the pendulum in the next half period would fall on top of the first circle. To explain this circular motion in a coordinate system fixed to the earth it would be necessary to invoke, in addition to gravity, the centrifugal force due to the circular motion and the Coriolis force due to the motion relative to the surface.

The above thoughts are pertinent to the consideration of what are called *inertial currents*. Water in the ocean that has been set in motion and is now drifting under its own inertia could be expected to keep deflecting to the right (NH) or left (SH) until it is moving in a circle (clockwise in the NH, counterclockwise in the SH) such that the centrifugal force away from the centre of the circle just balances the Coriolis force toward the centre. This is called the *inertial circle*. The time taken to complete the circle is the *inertial period*, and will be seen to equal one half pendulum day. If the water is moving at speed v in a circle of radius r, the centrifugal force away from the centre is v^2/r. Let f be the Coriolis parameter at the latitude φ ($f = 2\Omega \sin\varphi$), so that the Coriolis force toward the centre is fv. The balance of forces is therefore

(1.9.1) $fv = v^2/r$, whence $r = v/f$

The circumference of the inertial circle is thus $2\pi r = 2\pi v/f$, and the time taken to travel around the circumference is the inertial period, T_I, so

(1.9.2) $$T_I = 2\pi\frac{r}{v} = \frac{2\pi}{f} = \frac{2\pi}{2\Omega \sin\varphi}$$

Since $\Omega = \dfrac{2\pi}{24 \text{ hours}}$, $T_I = \dfrac{12}{\sin\varphi}$ hours

The inertial period is seen to be the half pendulum day as anticipated. This is a period that is frequently detected in ocean current measurements. At 45° latitude it is 17 h, at 30° it is 24 hours, and at 75° it is 12.4 hours (the same as the semidiurnal tidal period). From 1.9.1 the radius of the inertial circle is seen to be proportional to the current speed for a given latitude. At 45° latitude the radius for a 1 km/h current is 2.7 km, and at the pole it is 1.9 km. At the equator the radius is infinite, meaning there are no inertial circles there since the Coriolis force is zero. It should be noted that the motion in an inertial circle is not that of an eddy, and that all parts of the water are moving in the same direction at the same time in inertial motion. For those versed in carpentry a helpful analogy might be that of the movement of an orbital sanding plate (cf. inertial motion) versus the movement of a rotary sanding disk (cf. eddy motion).

1.10 Amphidromic systems

The word *amphidrome* is from the Greek for "a round race course," and describes a system in which wave crests propagate like the spokes of a wheel around a central *amphidromic point*, with wave amplitude increasing outward from zero at

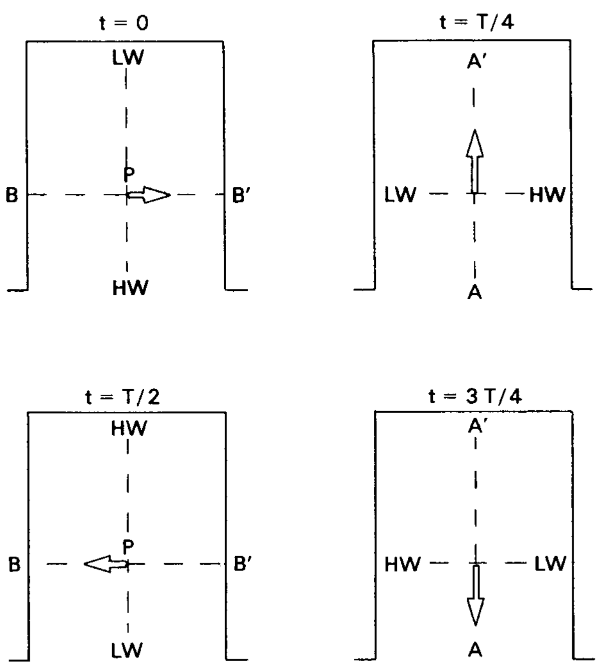


FIG. 10. Amphidromic system in a coastal embayment. Open arrows show the tidal streams (particle velocities).

the centre. Figure 10 illustrates the formation of an amphidromic system in a coastal embayment by the action of the Coriolis force on what would otherwise be a simple standing wave. Let the embayment be greater than a quarter wavelength long, so that in the absence of earth rotation there would be a nodal line across the embayment at BB', with high water (HW) at A coinciding with low water (LW) at A' and vice versa. Let us now add earth rotation and follow the oscillation through one period (T), starting with HW at A at time $t = 0$. From $t = 0$ to $t = T/2$, there is an axial flow through P from A to A', and the resulting Coriolis force to the right sets up a cross flow component from B to B' sufficient to create a surface slope that balances the Coriolis force. Since the axial flow is greatest at $t = T/4$, the surface slope across the embayment is also greatest then, giving a HW at B' and a LW at B at $t = T/4$. At $t = T/2$ HW is at A' and LW at A, as in the ordinary standing wave. At $t = 3T/4$ the axial flow through P from A' to A is maximum, and HW will be at B with LW at B', to provide the surface slope necessary to balance the Coriolis force on the outflowing water. Thus, in the Northern Hemisphere earth rotation can convert a simple standing wave in a basin into an amphidromic system (or amphidrome), in which the crest travels counterclockwise around the perimeter of the basin about a pivotal point, P, called the amphidromic point. The vertical amplitude is zero at P and the particle velocity reaches its maximum there, but now the particle velocity vector rotates counter-clockwise, tracing out an ellipse. The amplitudes of the wave at B and B' and of the particle velocity across the basin depend on the geometry and size of the basin and the length of the period of oscillation relative to that of the half pendulum day. The origin and nature of amphidromes in the open ocean are less simple than those described above, and sometimes the sense of rotation is opposite to that in an embayment. Figure 29 shows the amphidromic system of the semidiurnal tide wave in the Gulf of St. Lawrence, and Fig. 30 shows an amphidrome of the diurnal tide wave in the Atlantic Ocean off Nova Scotia.

1.11 Tides and tidal streams

Since the tide propagates as a set of long waves in the ocean, much of the character of its vertical and horizontal motion has been revealed in the preceding consideration of long waves. The terms defined in section 1.2 to describe the characteristics of a wave are also applied to tides, but some special tidal terms are used as well. The definitions given here conform as closely as possible to common usage in Canadian tidal literature. In a tide wave the horizontal motion, i.e. the particle velocity, is called the *tidal stream*. The vertical tide is said to *rise* and *fall*, and the tidal stream is said to *flood* and *ebb*. If the tide is progressive, the flood direction is that of the wave propagation: if the tide is a standing wave, the flood direction is inland or toward the coast, i.e. "upstream." The *flow* is the net horizontal motion of the water at a given time from whatever causes. The single word "current" is frequently used synonymously with "flow"," but the term *residual current* is used for the portion of the flow not accounted for by the tidal streams. A tidal stream is *rectilinear* if it flows back and forth in a straight line, and is *rotary* if its velocity vector traces out an ellipse. Except in restricted coastal passages, most tidal streams are rotary, although the shape of the ellipse and the direction of rotation may vary. The ellipse traced out by a tidal stream vector is called the *tidal ellipse*. *Slack water* refers to zero flow in a tidal regime. The *stand of the tide* is the interval around high or low water in which there is little change of water level: this need not coincide with slack water.

In a purely progressive surface tide, maximum flood occurs at HW, maximum ebb occurs at LW, and slack water occurs at mid-tide rising and falling. In a purely standing surface tide, the slack waters occur at HW and at LW, maximum flood occurs at mid-tide rising, and maximum ebb occurs at mid-tide falling. This follows from the discussion in section 1.5, and is illustrated in Fig. 5 for long waves in general. Except for some frictional effect near the bottom, the tidal streams associated with a surface tide are the same from top to bottom. If tidal streams are observed to vary in speed, phase or direction over the water column, the presence of an internal tide is indicated. The average tidal stream in such a case belongs to the surface tide, and the departures at various depths from this average are the tidal streams belonging to the internal tide. This situation presents the possibility for slack water to occur at different times at different depths. Figure 11 illustrates various flow patterns that may result from the vector addition of a residual current and a rectilinear or a rotary tidal stream. It is seen that a rectilinear tidal stream experiences slack water twice during each period (Fig. 11a) unless it is accompanied by (1) a residual current in the same direction but with speed greater than the tidal stream amplitude, in which case the flow is unidirectional with varying speed (Fig. 11b), or (2) a residual current in a different direction from that of the tidal stream, in which case the flow changes direction through a small angle (Fig. 11c). It is also apparent from Fig. 11 that a rotary tidal stream rarely experiences slack water, but that its direction changes through 360° in each cycle (Fig. 11d) unless the speed of the residual current exceeds or equals the amplitude of the tidal stream in that direction (Fig. 11e and f). In the latter cases, the direction of the flow swings back and forth through an angle less than or equal to 180°.

Since the observed tide consists not of a single wave, but of the superposition of many tide waves of different frequency and amplitude, it will never fit exactly any of our simple descriptions. Because of this, we cannot expect the heights of successive HWs or of successive LWs to be identical, even when they occur in the same day. Thus, the two HWs and two LWs occurring in the same day are designated as *higher and lower high water* (HHW and LHW), and *higher and lower low water* (HLW and LLW). It is likewise only the tidal stream associated with a single frequency tide wave that traces a perfect tidal ellipse. The composite tidal stream each day traces a path more closely resembling a double spiral, with no two days' patterns identical. Also, no tide is ever a purely progressive or a purely standing wave, so that slack water should not be expected to occur at the same interval before HW or LW at all locations.

1.12. Shallow-water effects

One of our assumptions in the discussion of long waves of sinusoidal form was that the amplitude was much less than the depth. When a tide propagates into shallow water, this assumption may no longer be valid, and, as might be expected, the wave form is distorted from its sinusoidal form. In such shallow water the crest is found to propagate faster than the trough, producing a steeper rise and a more gradual fall of the water level as the tide wave passes. Figure 12 demonstrates this effect on the St. Lawrence River tide between Neuville and Trois Rivières. The outflow of the river and the bottom friction contribute to the distortion of the wave. The tide in this part of the St. Lawrence River is attenuated by friction as it progresses upstream, and is not reflected to produce a standing wave.

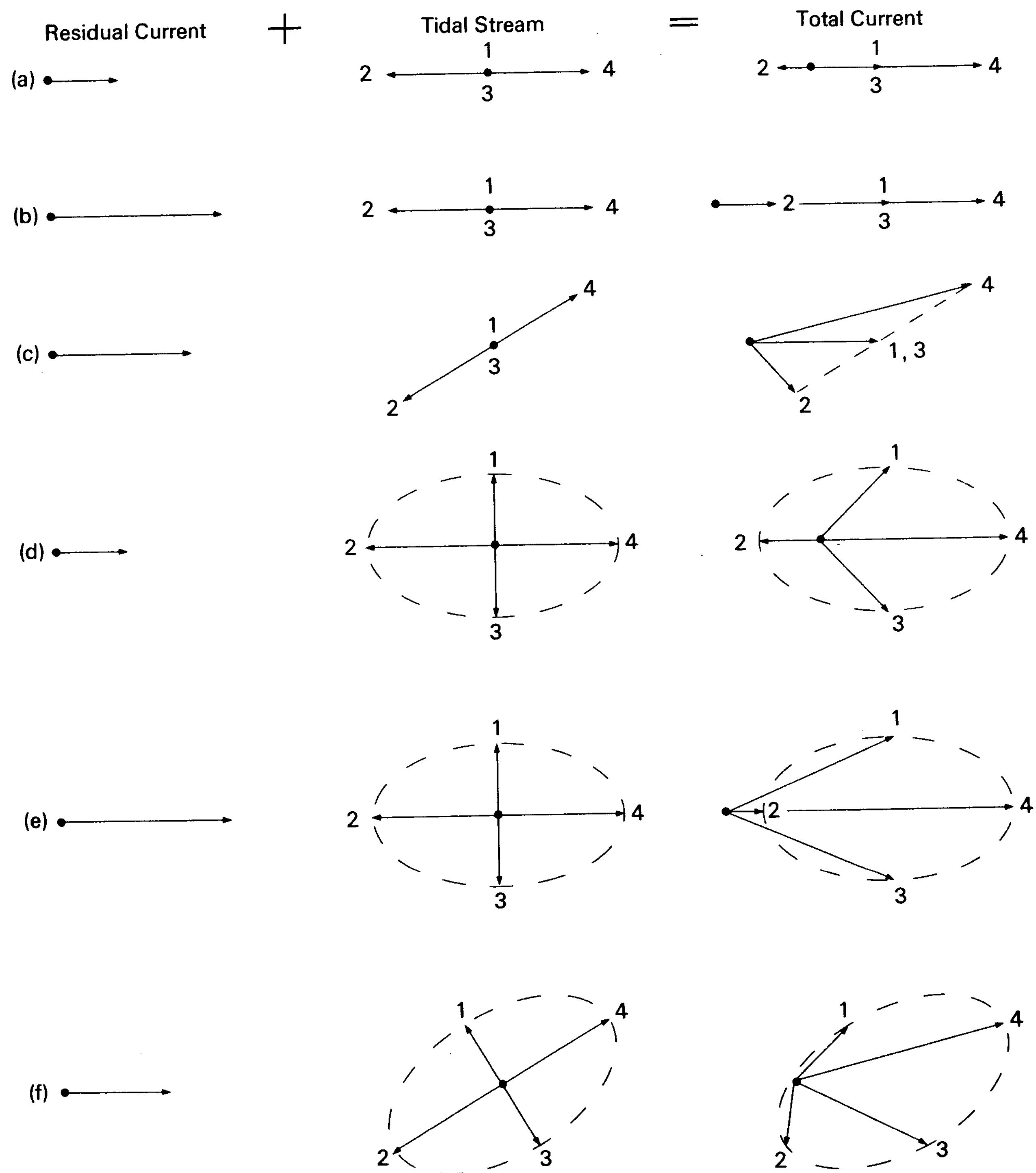


FIG. 11. Flow patterns resulting from combination of various residual currents with rectilinear and rotary tidal streams.

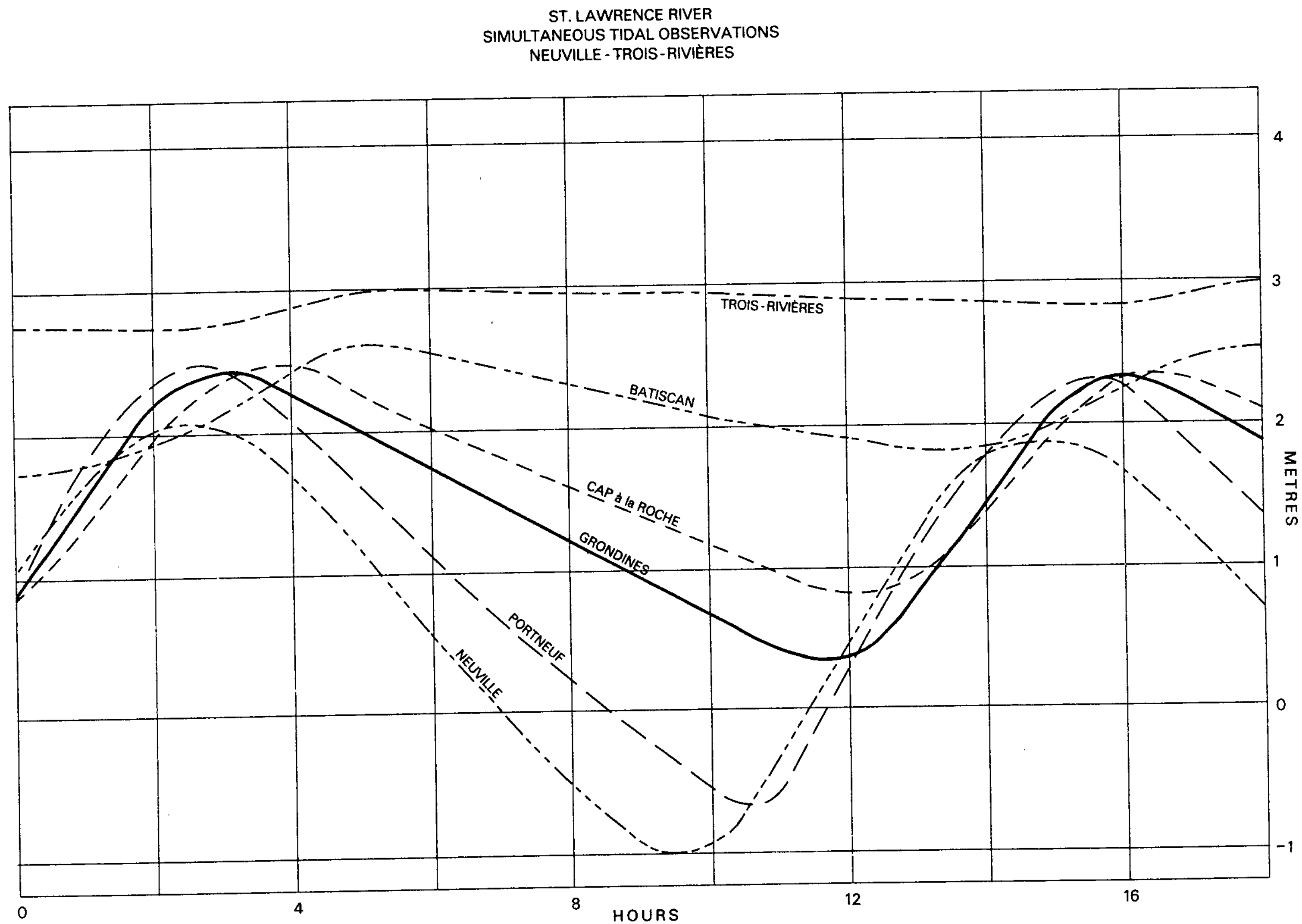


Fig. 12. Shallow water distortion of tide wave in St. Lawrence River, between Neuville and Trois Rivières. (from figure 13 of *Tides in Canadian Waters*, by G. Dohler).

Tides in the open ocean are usually of much smaller amplitude than those along the coast. As mentioned earlier, this is partly due to amplification by reflection and resonance. It is, however, more generally the result of *shoaling:* as the wave propagates into shallower water, its wave speed decreases and the energy contained between crests is compressed both into a smaller depth and a shorter wavelength. The tide height and the tidal stream strength must increase accordingly. If, in addition, the tide propagates into an inlet whose width diminishes toward the head, the wave energy is further compressed laterally. This effect, called *funneling*, also causes the tide height to increase.

Sometimes the front of the rising tide propagates up a river as a *bore,* a churning and tumbling wall of water advancing up the river not unlike a breaking surf riding up a beach. Creation of a bore requires a large rise of tide at the mouth of the river, some sandbars, or other restrictions at the entrance to impede the initial advance of the tide, and a shallow and gently sloping river bed. Simply stated, the water cannot spread uniformly over the vast shallow interior area fast enough to match the rapid rise at the entrance. Friction at the base of the advancing front, plus resistance from the last of the ebb flow still leaving the river, causes the top of the advancing front to tumble forward, sometimes giving the bore the appearance of a travelling waterfall. There are spectacular bores a metre or more high in several rivers and estuaries of the world. The best known bore in Canada is that in the Petitcodiac River near Moncton, N.B., but there is another in the Shubenacadie River and in the Salmon river near Truro, N.S., all driven by the large Bay of Fundy tides. These are impressive (about a metre) only at the time of the highest monthly tides, and may be no more than a large ripple during the smallest tides.

The *reversing falls* near the mouth of the St. John River at Saint John, N.B. is also caused by the large Bay of Fundy tides and the configuration of the river. A narrow gorge at Saint John separates the outer harbour from a large inner basin. When the tide is rising most rapidly outside, water cannot pass quickly enough through the gorge to raise the level of the inner basin at the same rate, so on this stage of the tide the water races in through the gorge, dropping several metres over the length of the gorge. When the outside tide is falling most rapidly, the situation is reversed, and the water races out through the gorge in the opposite direction, again dropping several metres in surface elevation. Twice during each tidal cycle, when the water levels inside and out are the same, the water in the gorge is placid and navigable. The surface of the water in the gorge near the peak flows is violently agitated and the velocity of flow is too rapid and turbulent to permit navigation through the gorge. This phenomenon is called a *tide race* in other less notorious situations.

A *tide rip* or *overfall* is an area of breaking waves or violent surface agitation that may occur at certain stages of the tide in the presence of strong tidal flow. They may be caused by a rapid flow over an irregular bottom, by the conjunction of two opposing flows, or by the piling up of waves or swell against an oppositely directed tidal flow. If waves run up against a current, the wave form and the wave energy are compressed into a shorter wavelength, causing a growth and steepening of the waves. If the current is strong enough, the waves may steepen to the point of breaking, and dissipate their energy in a wild fury at sea. Violent tide rips may be formed in this way.

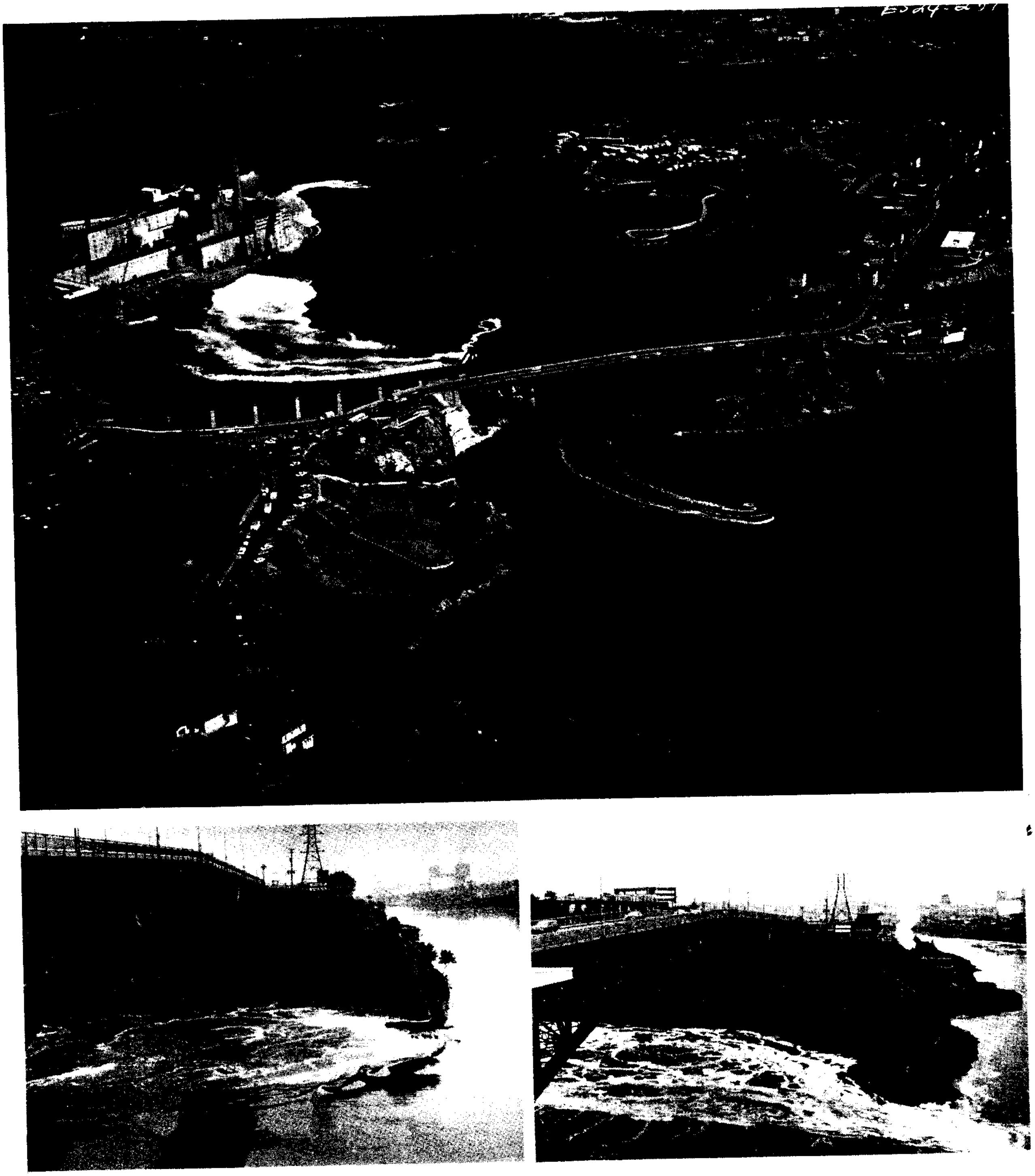


PLATE 6. The Reversing Falls at Saint John, New Brunswick, at the mouth of the St. John River. The upper photo is an aerial view at slack water, showing the inner basin, the outer harbour, and the bridge over the gorge that separates them. Lower left shows the inflow through the gorge at high water in the outer harbour (7.6 m above chart datum at time of photo). Lower right shows the outflow through the gorge at low water in the outer harbour (0.9 m above chart datum at time of photo). The recorded extreme high and low waters at Saint John are 9.0 and –0.4 m, respectively, above chart datum, and at these times the flows would have been correspondingly greater. (Upper photo by Lockwood Survey, NFB Phototeque, 1966; Lower photos by D.G. Mitchell, Canadian Hydrographic Service, 1963.)

PLATE 7. (*Upper*) Tidal bore on the Petitcodiac River at Moncton, New Brunswick. (Photo by D.G. Mitchell, Canadian Hydrographic Service, 1960.); (*Lower*) Tidal bore on the Salmon River, near Truro, Nova Scotia. (Photo by F.G. Barber, Ocean Science and Surveys, DFO, 1982.)