CHAPTER 3

Tidal Analysis and Prediction

3.1 Introduction

We have demonstrated in Chapter 2 how to identify the fundamental tidal frequencies present in the spectrum of the equilibrium tide, and have speculated that these should be the fundamental frequencies present in any actual tidal record. Basically, tidal analysis consists of identifying in a tidal record the amplitudes of all the important harmonic constituents and their phaselags with respect to the phases of the corresponding constituents in the equilibrium tide. Tidal prediction consists of recombining these constituents in the proper phase relation to the corresponding constituents in the equilibrium tide at the desired times. Both analysis and prediction require a knowledge of the phases of the harmonic constituents in the equilibrium tide at specified times, which may be extracted from tables or calculated by formulae involving the astronomical parameters. Although it is usually easier to take things apart than it is to put them back together again, the principles of tidal prediction are much simpler than are those of tidal analysis. Nevertheless, we will start with a consideration of tidal analysis, the breaking down of a tidal record into its component parts. Initially, a look at the Fourier theorem will be helpful.

3.2 The Fourier Theorem

This theorem states that if $F(t)$, a function of the variable t, is defined over the range from $t = -T/2$ to $t = T/2$, then $F(t)$ can be expressed as a constant plus an infinite series of sinusoids of frequencies (or wave numbers, if t is thought of as a distance) $1/T$, $2/T$, $3T, \ldots i/T \ldots$ etc. The sinusoidal oscillation with frequency $1/T$ is called the *fundamental*, and the oscillations whose frequencies are multiples of $1/T$ are called *harmonics* of the fundamental. Every harmonic that is added to the series increases the precision to which it reproduces $F(t)$ in the range $-T/2$ to $T/2$. Outside this range, the series of sinusoids would produce the same image of $F(t)$ between $T/2$ and $3T/2$, and repeat it again and again for every interval of length T. Unless $F(t)$ is believed to be periodic with period T over all values of t, these repetitions are simply ignored, and the series of sinusoids is used only to reproduce values of $F(t)$ within the defined range $-T/2$ to $T/2$. Instructions are also included in the theorem for the evaluation of the terms in the series. The constant term is simply the average value of $F(t)$ over the defined range. Evaluation of the fundamental or one of the harmonics involves multiplying every point in $F(t)$ by the sine and by the cosine of that harmonic frequency times t, and forming the average of these products over T. The Fourier theorem is stated much more compactly in mathematical form as follows:

If $F(t)$ is defined between $-T/2$ and $T/2$, then

$$(3.2.1) \quad F(t) = H_o + \sum_{i=1}^{\infty} H_i \cos\left(2\pi i \frac{t}{T} - \theta_i\right)$$

where H_o = avg. over T of $F(t)$,

$$H_i \sin\theta_i = \text{avg. over } T \text{ of } 2F(t)\sin 2\pi i\frac{t}{T}$$

$$H_i \cos\theta_i = \text{avg. over } T \text{ of } 2F(t)\cos 2\pi i\frac{t}{T}$$

It would be convenient indeed if the tidal cycle repeated itself exactly at regular intervals such as a month or a year, because then a Fourier series could be formed as described above to provide predictions for all time, with no regard to further tidal theory. Certainly a Fourier series can be formed to reproduce any finite tidal record to any desired accuracy, but since tides do not repeat exactly after any known interval, the series could not be used to predict values for any time outside the record, and would be of little value. So what is the pertinence of the Fourier theorem to tidal analysis? If, in 3.2.1, we allow T to become infinite, the fundamental frequency becomes infinitesimal, and the frequency interval between harmonics also becomes infinitesimal; i.e. all frequencies become candidates for inclusion in the Fourier series. Our knowledge of the frequencies present in the equilibrium tide tells us to look only at these frequencies out of the Fourier spectrum of all possible frequencies. The last two equations of 3.2.1 are then used to estimate the amplitudes and phaselags of the tidal constituents from observed tidal records. The tidal records are not infinitely long, however, and some ingenuity is required in forming meaningful averages for the expressions in 3.2.1 from short tidal records. This is the challenge of tidal harmonic analysis.

3.3 Harmonic analysis of tides

The harmonic method of analysis treats every tidal record as consisting of a sum of *harmonic constituents* of known frequency plus non-tidal "noise." This may be expressed as

$$(3.3.1)\quad h(t) = \sum_{i=0}^{n} H_i' \cos (E_i' - g_i) + \text{"noise"}$$
$$= H_0' + \sum_{i=1}^{n} H_i' (\cos E_i' \cos g_i + \sin E_i' \sin g_i) + \text{"noise"}$$

in which $h(t)$ is the instantaneous height, H_i' is the amplitude of the ith constituent, E_i' is the phase of the equilibrium constituent at Greenwich at Greenwich Mean Time (GMT) numerically equal to the local observation time, and g_i is the phaselag of the constituent behind the Greenwich phase, E_i'. It is important to note that the Greenwich phase used here is the actual phase of the equilibrium constituent at Greenwich only if the observations are recorded in GMT. If they are recorded in a time zone z hours west of Greenwich, then E_i' is the phase of the constituent at Greenwich z hours earlier. This may seem confusing at first, but it avoids the need to convert observation times into GMT, and the choice of a reference phase can be quite arbitrary as long as it advances at the proper speed and is applied consistently in all calculations. It is, however, necessary to record the time zone carefully along with the results of any tidal analysis to assure consistency in phase reference. The significance of the primes on H_i' and E_i' will become apparent later. The subscript, i, is simply a counter for the n harmonic constituents considered necessary to represent the tidal signal adequately; H_o' represents the average water level during the record (zero frequency, $E_o' = g_o = 0$).

The aim of the harmonic analysis is to determine the values of all the H_i' and the g_i. H_o' is simply the average of all the observations, and is usually denoted as Z_o in tidal terminology. If each value of $h(t)$ is multiplied by $\cos E_1'$ and the average taken over the length of the record, 3.3.1 gives

$$(3.3.2)\quad \text{avg. } [(\cos E_1')h(t) = H_1' \cos g_1 (\cos^2 E_1') + H_2' \cos g_2 (\cos E_1' \cos E_2') + H_1' \sin g_1(\cos E_1' \sin E_1') + H_2' \sin g_2(\cos E_1' \sin E_2') + \text{etc.} + (\cos E_1')(\text{"noise"})]$$

and, if each value of $h(t)$ is multiplied by $\sin E_1'$, the average gives

$$(3.3.3)\quad \text{avg. } [(\sin E_1')\, h(t) = H_1' \sin g_1(\sin^2 E_1') + H_2 \sin g_2 (\sin E_1' \sin E_2') + H_1' \cos g_1(\sin E_1' \cos E_1') + H_2' \cos g_2(\sin E_1' \cos E_2') + \text{etc.} + (\sin E_1')(\text{"noise"})]$$

The "noise" terms are considered to average to zero, their signal being assumed to be random with respect to tidal frequencies. If all n constituents could complete an exact number of cycles in the same length of record, all of the averaged coefficients in 3.3.2 and 3.3.3 would be zero except for $\cos^2 E_1'$ and $\sin^2 E_1'$, whose average values would be 0.5. This would give the simple Fourier solution of 3.2.1, namely

$$(3.3.4)\quad H_1' \sin g_1 = \text{avg. of } 2h(t) \sin E_1'$$
$$H_1' \cos g_1 = \text{avg. of } 2\, h(t) \cos E_1'$$

Of course the tidal constituents cannot all complete an exact number of cycles in the same length of record, and we must contend with the residual terms in 3.3.2 and 3.3.3 instead of assuming the enticingly simple solution of 3.3.4. Repeating the above process of multiplication by the sine and cosine of the E_i' and averaging for the other n–1 constituents completes a set of $2n$ equations in the $2n$ unknowns (the H_i' *sin* g_i and the $H_i' \cos g_i$). As a matter of interest, these are the same $2n$ "normal equations" that would have been produced if the problem had been tackled by the "method of least squares," so their solution for the H_i' and g_i gives a best fit to the data in a "least squares" sense.

While the generation of the $4n^2$ coefficients and the solution of the $2n$ simultaneous equations in $2n$ unknowns is feasible with today's high-speed, large-memory electronic computers, such was not always so in the days of manual computation. One simplification that is frequently used for manual computation is to replace the sine and cosine multipliers by the "box-car" functions that equal +1 when the corresponding sine or cosine are positive, equal −1 when they are negative, and equal zero when the sine or cosine are zero. Figure 23 illustrates how multiplication by the box-car equivalent of the sine or cosine function produces an average contribution of $2/\pi$ times the corresponding (sine or cosine) component of a constituent of the same frequency, zero times the complementary (cosine or sine) component of a constituent of the same frequency, and zero times both the cosine and sine components of a constituent of exactly twice the frequency. The principle by which the pure sine and cosine multipliers sort out the coefficients of 3.3.3 and 3.3.4 is the same as that by which their box-car analogues worked in Fig. 23. The box-car multipliers, however, are most effective in separating one species of tide from another (diurnal, semi-diurnal, etc.) but for separation of constituents of the same species, the pure sine and cosine multipliers produce a better-behaved set of coefficients.

3.4 Nineteen-year modulation of lunar constituents

The moon's orbit about the earth is in a plane always inclined at approximately 5° to the plane of the earth's orbit about the sun (the ecliptic), but the line of intersection of the two planes rotates once every 18.6 years about the pole of the ecliptic. Figure 24 shows a celestial sphere (a sphere of infinite radius, with the earth's centre as its centre) onto which are projected the earth's equator and its polar axis, the plane of the ecliptic and its axis, and the plane of the moon's orbit and its axis. The

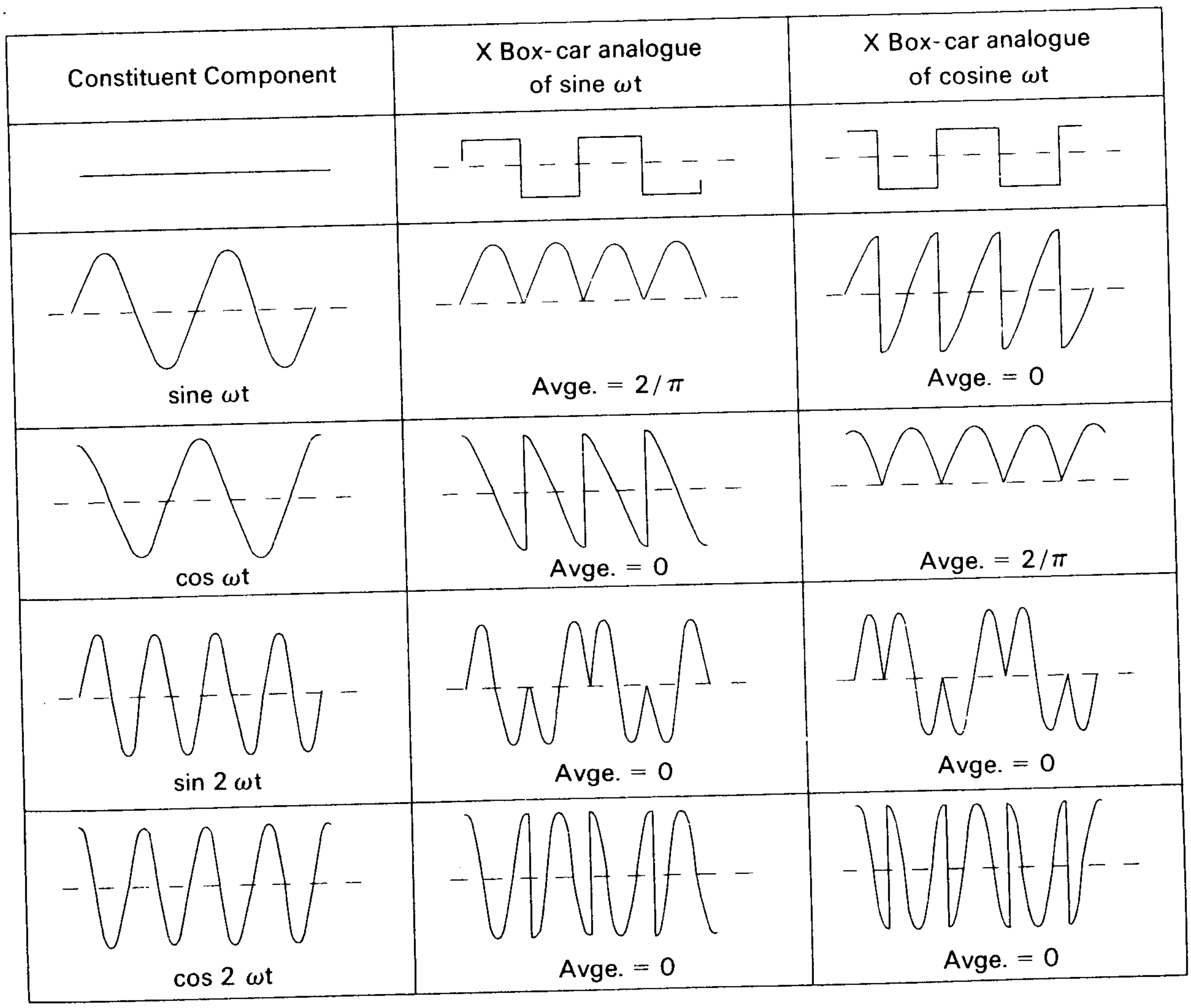

FIG. 23. Numerical filtering of tidal constituents by multiplication of signal by box-car functions.

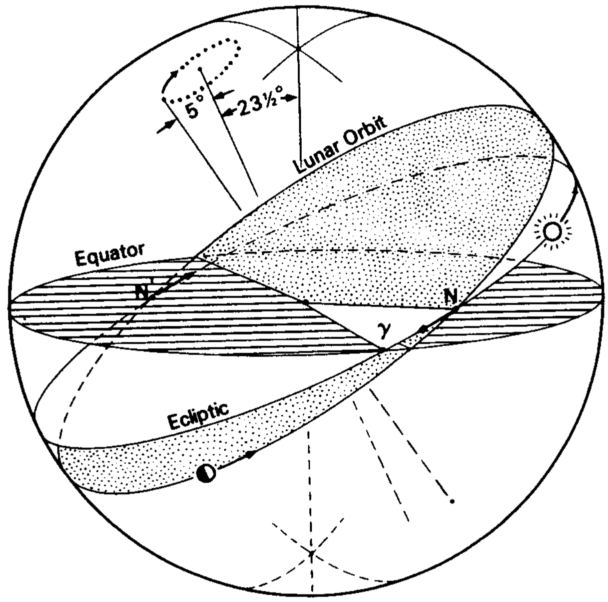

FIG. 24. Celestial sphere, showing equator, ecliptic, lunar orbit and regression of moon's ascending node, N.

spherical screen of a planetarium is a model of a portion of a celestial sphere. The line NN' is the intersection of the moon's orbital plane with the ecliptic, and N and N' are referred to as "nodes." N is the "ascending node," since the moon is moving from south to north of the ecliptic as it passes N. As the axis of the moon's orbit rotates about the axis of the ecliptic, tracing out the 5° cone once every 18.6 yr, the point N moves around the ecliptic from east to west in the same period. Since the moon travels its orbit in the opposite sense (west to east), this is referred as the "regression of the moon's ascending node," and it has a major influence on the moon's declination over the approximate 19-year period. The inclination of the ecliptic to the equator is 23½°, so over the course of a year the sun changes declination between 23½° north in summer and 23½° south in winter. Since the moon's orbit is inclined at 5° to the ecliptic, the moon's declination may change over the course of a month between 28½° (i.e. 23½° + 5°) north and south during one part of the 19-year cycle, and between only 18½° (i.e. 23½° − 5°) 9½ years later in the cycle. The moon's maximum monthly swing in declination occurs when its ascending node, N, coincides with the vernal equinox, and its minimum swing occurs when its ascending node coincides with the autumnal equinox. The vernal equinox (γ) is the point at which the sun crosses the equator on its way north, and the autumnal equinox is the point at which it crosses it on its way south.

The regression of the moon's ascending node has the effect of modulating both the amplitude and the phase of the lunar tidal constituents in a 19-year period. Because the period is so long, it is assumed that the modulation of the constituents in the real tide is the same as that of the equilibrium constituents. The amplitude modulation is represented by a *nodal factor*, f, which varies about a mean value of unity over the period of 18.6 years. The phase modulation is represented by a *nodal correction*, u, which varies about a mean value of zero over the same period. There is no nodal modulation of the solar constituents, and the f and u values are different for each lunar constituent. Values of the nodal parameters are tabulated, and may also be computed from formulae involving the astronomical variables. f of K_2 varies between about 0.75 and 1.30, while f of M_2 varies only between 0.96 and 1.04. u of K_2 varies between plus and minus 17°, while u of M_2 varies only between plus and minus 2°.

To conform with the above, the equilibrium phase, E_i', used in the harmonic analysis must be the phase of the mean equilibrium constituent, E_i, plus the nodal correction, u_i, for that constituent at that time. The amplitude, H_i', that results from the analysis will be the amplitude of the mean constituent, H_i, times the nodal factor, f_i, for that constituent at that time. Thus, in section 3.3

$$E_i' = E_i + u_i, \text{ and } H_i' = f_i H_i$$

The tidal constants that are retained from the analysis are the amplitudes of the mean constituents ($H_i = H_i'/f_i$) and the phaselags of the mean constituents (g_i).

3.5 Shallow-water constituents

Section 1.12 discusses, and Fig. 12 illustrates, the distortion of a tide wave as it travels in shallow water. The Fourier theorem suggests that this distortion could be represented by adding harmonics of the fundamental tidal frequencies, a procedure that is attractive because it is compatible with the methods of harmonic analysis and prediction. This, then, is the origin of the *shallow-water constituents* (sometimes called "over-tides"). They are introduced as a mathematical convenience to represent distortion of the tide wave, and do not arise directly from the tidal forces. As an example, M_6 is the second harmonic of the fundamental tidal constituent M_2, with three times its angular speed. The most common shallow-water constituents are the quarter-diurnals M_4 and MS_4, with frequencies twice that of M_2 and the sum of those of M_2 and S_2, respectively. Their combination produces a quarter-diurnal tide whose amplitude is modulated at their difference frequency, which is the same as the modulation frequency of the semidiurnal tide produced by the combination of M_2 and S_2. This part of the quarter-diurnal tide is thus able to increase and decrease in the same spring-neap cycle as the part of the semidiurnal tide produced by the combination of M_2 and S_2. Figure 25 shows graphically the distortion produced in a fundamental constituent by

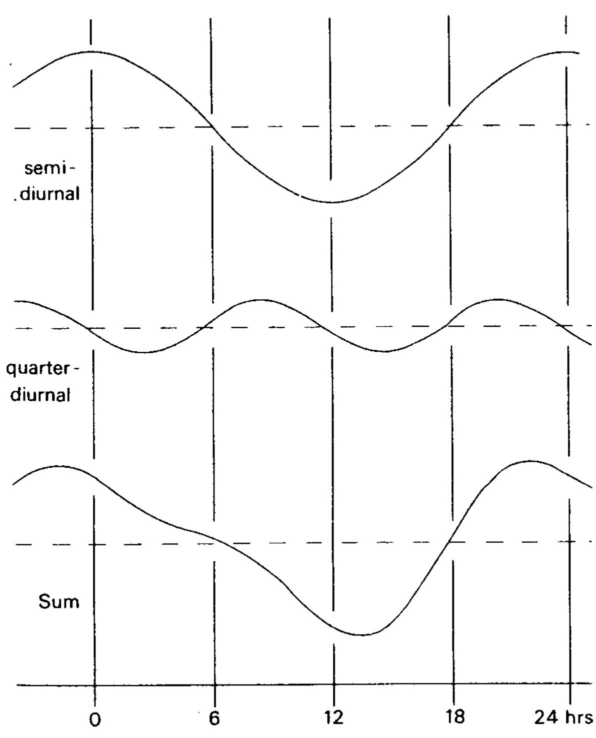

FIG. 25. Representation of shallow water distortion by addition of a first harmonic to the fundamental frequency.

the addition of a first harmonic. This may be compared to the distortion of the semidiurnal tide in the St. Lawrence River, shown in Fig. 12. The shallow-water constituents can be included in the harmonic analysis procedure described in section 3.3.

3.6 Record length and sampling interval

Theoretically, if a water level record contained nothing but a pure tidal signal consisting of the contributions from n pure tidal constituents, the amplitudes and phaselags of the n constituents could be determined from almost any set of $2n$ observation points. Of course, nothing in this world is so pure, and tidal records are contaminated with "noise," both of meteorological and observational origin. This is why it is necessary to rely on statistical averaging and filtering of long tidal records, as described in section 3.3, to resolve the tidal signal from the background noise and to distinguish the individual harmonic constituents. In practice, the lowest frequency constituent that could possibly be distinguished in a tidal record is one whose period equals the length of the record, and the highest frequency constituent is one whose period is twice the sampling interval. Whether two neighbouring constituents can be separated from each other in an analysis depends both on the difference in their frequency and the length of the record. The "Rayleigh criterion" for the separation of two constituents requires that they should change phase with respect to each other by at least 360° during the record period. If n_1 and n_2 are the constituents' angular speeds in degrees per hour and T is the record length in hours, the Rayleigh criterion for separation is

(3.6.1) $(n_1 - n_2)T \geq 360°$

Sometimes, if a record seems relatively free of noise, this criterion might be relaxed, and the right side of 3.6.1 made 180°. Diurnal constituents could be separated from semidiurnal constituents on the basis of a single day's record, but to separate constituents of the same species requires a much longer record. As an example, consider the length of record required to permit the separation of constituents M_2 and S_2. From Appendix A we see that their difference in speed is 1.016°/h, so that 3.6.1 gives the length of record necessary to separate them as $T = 360/1.016 = 354$ h, or 14.8 days. But to separate N_2 from M_2 would, by the same reasoning, require a record length of $T = 360/0.544 = 662$ h, or 27.6 days; and to separate K_2 from S_2 would require $T = 360/0.082 = 4390$ h, or 183 days. It should not be surprising that the separation periods turn out to be the basic astronomical periods or fractions of them, since it was from these that the constituents inherited their frequencies.

When a record is too short to allow separation of all the constituents that are known to contribute significantly to the tide in that region, relationships must be assumed between the inseparable constituents. These are called *regional relations*, because the justification for their adoption is that waves of such nearly equal frequencies must behave very similarly, and so maintain very nearly the same relation to each other over a large region. The ratio of amplitudes and the difference of phaselags of the two constituents are therefore assumed to be the same as those observed at the nearest comparable location for which a more complete analysis is available. In any tidal analysis, every harmonic constituent must be (a) included directly in the analysis, (b) allowed for through regional relations with other constituents, or (c) omitted because it is known to be negligible in the region of observation. The longer the tidal record, the greater is the number of constituents that can be analysed, and the higher is the accuracy of their determination. For temporary water level gauges installed in tidal waters, a minimum recommended length of record is 1 month.

Tidal analyses are usually carried out on data that has been read at hourly intervals from the original record. A sampling interval of 1 h is short enough to detect the highest frequency constituents of interest in most tidal work. However, if higher frequencies are seen to be present, even though they may not be of direct interest, they should be smoothed out of the record before the hourly samples are taken. This is to avoid the process known as *aliasing*, by which frequencies higher than the sampling frequency may masquerade as lower frequencies. Figure 26 illustrates the principle of aliasing. It is usually more important to smooth the samples from a short record than from a long one, because the aliased signal is likely random with respect to the tidal signal, and so can be adequately eliminated in the averaging and filtering of the long analysis. Today's computer programs for tidal analysis are sufficiently flexible to accept data that has been sampled at irregular intervals, and even data that has gaps of several days in it. In fact, two records taken at different times may be more valuable than a single continuous record of their combined length. For example, 1 day of record at spring tide and 1 day of record at neap tide could permit separation of M_2 and S_2, whereas 2 days of continuous record could not. A single continuous record covering the whole period from spring to neap tide would, of course, be more valuable still.

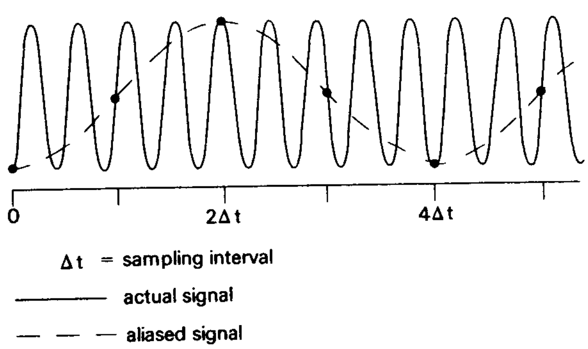

Fig. 26. "Aliasing" to a lower frequency by an oscillation whose period is less than the sampling interval.

3.7 Harmonic analysis of tidal streams

The only difference between the analysis of a water level record and the analysis of a current meter record is that the current speed and direction data must first be resolved into two mutually perpendicular horizontal vector components. Two separate harmonic analyses are then performed, one on each time series of component speeds. The choice of the two component directions is arbitrary, as long as it is well recorded what they are. A common choice, and one that is recommended, is that of the true azimuths North and East. If the current observations were taken in a well defined channel or passage, the directions along and across the channel might be chosen for the components. The result of the analysis of a current record is two sets of harmonic constants (amplitudes and phaselags), one for each component direction. Even though the two directions are known to be perpendicular to each other, it is important to quote both of them, to avoid a possible 180° uncertainty. Sometimes the constants for each harmonic constituent are used to calculate that constituent's tidal ellipse (see section 1.11). The ellipse may be described by the direction of its major axis, the amplitude and phaselag of the tidal stream component in that direction, the tidal stream amplitude in the minor axis direction, and a statement that the sense of rotation of the stream is clockwise or counterclockwise. In tidal ellipse form, the phaselag on the minor axis is always 90° different from that on the major axis. The sense of rotation of the stream tells how the difference should be applied. There is no objection, of course, to stating explicitly the direction of the minor axis and the phaselag as well as the amplitude for that direction. The tidal ellipse form for the results of a tidal stream analysis has some advantage if it is wished to display the individual harmonic constituents graphically, but it has the significant disadvantage that the stream components are resolved in different directions for each harmonic constituent.

Tidal streams reflect the presence of internal tides as well as of surface tides, and, because of the properties of internal waves discussed in section 1.7, tidal streams may vary in depth, may vary seasonally with changes in stratification, and may vary spatially in a pattern different from that of the surface tide. For these reasons, the coherence between tidal stream records taken on different moorings at different times is usually poor, and the records should not be combined into a single analysis. Even if a long current record is obtained from a single mooring, it is usually wise to break the record up into one month lengths for separate analysis. Care should also be used in applying regional relations for the constituents in tidal stream analysis, because of the greater variability in tidal streams than in surface tides in the same region.

3.8 Harmonic method of tidal prediction

Prediction of the tidal height at any desired time, t, involves summing the contributions from all the important harmonic constituents in their proper phase for that time, and adding their sum to the mean water level, Z_o. Expressed mathematically, this is

$$(3.8.1)\quad h(t) = Z_o + \sum_{i=1}^{n} f_i H_i \cos(E_i + u_i - g_i)$$

where the symbols have the same meanings as in sections 3.3 and 3.4. The nodal parameters, f and u, and the Greenwich phase of each equilibrium constituent, E, may be obtained for time t either from tables or from formulae involving known astronomical parameters. According to the convention explained in section 3.3, the values of E must be obtained for GMT numerically equal to t, for t in the same time zone to which the phaselags, g, refer. Values for the constituent amplitudes and phaselags, H and g, come from a harmonic analysis of tidal data previously recorded at that location, or, perhaps, from interpolation between known values at nearby locations. There is no model of world tides that is accurate or detailed enough to provide local tidal predictions from first principles. Previous observation of the tide in a region is a prerequisite to its prediction. The word "prediction" as used here includes calculation of tidal heights for past times (hindcasting) as well as for future times (forecasting).

The prediction of tidal streams is basically the same as the prediction of tides, except that predictions must be made separately in the two component directions, and then combined vectorially to give speed and direction. In 3.8.1, Z_o would be the component of the steady current in that direction. Many requirements for current and tidal stream predictions are for channels in which the flow is nearly rectilinear along the axis of the channel. In these cases it is possible to limit the predictions to one component direction, that along the axis of the channel.

3.9 Prediction of tidal and current extrema

The prediction of the times and heights of high and low water (HW and LW) is usually done by generating a series of predicted heights at hourly intervals and scanning it for changes in trend. When a change in trend (e.g. from increasing to decreasing height) occurs, intermediate predictions are inserted until the extreme height at which the trend reverses is located within a narrow interval. The time and height of the extremum (HW or LW) are then interpolated in the interval. Ingenious mechanical analogue apparatus was used in the past to generate the time series, and the scanning was done visually. Today, the task is accomplished almost exclusively by electronic digital computers programmed to perform all the steps.

Most locations for which current predictions are made are in channels or passages where the current is nearly rectilinear along the axis of the channel. Predictions are usually published only for the times and speeds of maximum flood and ebb and for the times of slack water. Using the set of harmonic constants for the component direction along the channel, a time series of current speeds is generated, with the positive sign for the flood direction and the negative sign for ebb. This series can be scanned to identify maxima and minima in the absolute value of the speed (i.e. without regard to sign). When the maxima are found, the sign of the speed at those times identifies them as floods (+) or ebbs (−). The minima usually occur at zero speed, and thus give the times of slack water. However, when there is a large residual current, the tidal streams may not be large enough to cause a reversal in the flow, and the minima may not be zero. In these cases the minima are identified as minimum flood or ebb according to the sign of the speed, and there is no slack water and no "time of turn."

For locations in which the currents are not rectilinear, a time series of current vectors may be produced from the two component sets of harmonic constants and scanned for maxima and minima in the magnitude (speed) of the vector. Since the concept of flood and ebb is inexact when the current is not rectilinear, the extrema should be identified by time, speed, and direction. In quoting the direction of a current the convention used is opposite to that used for wind direction: the direction of a current is the direction toward which it is flowing. Figure 11 illustrates some of the many patterns that may result from the combination of a steady current with a rectilinear or rotary tidal stream. If there is a large diurnal inequality in the tidal stream, the patterns could be even more variable.

3.10 Cotidal charts

Cotidal charts of the major harmonic constituents of the tide are frequently constructed to illustrate their different propagation patterns and to assist in the interpolation of the harmonic constants at locations where no observations exist. A cotidal chart consists of a set of *co-phase lines* and a set of *co-range lines* drawn on a suitable chart. Each co-phase line traces out the locus of points at which the constituent has a particular phaselag, and each co-range line traces out the locus of points at which it has a particular amplitude. Provided sufficient information is available, there is no limit to how large an area may be covered by a cotidal chart for a single harmonic constituent, and some have been constructed covering the whole world ocean. Figures 27 and 28 show cotidal charts of Hudson and James bays for the constituents M_2 and K_1, respectively. The marked difference in the two reflects the fact that the basin responds differently to waves of different frequencies. Charts of constituents within the same species of tide usually resemble each other quite closely over fairly large regions, and may be combined into composite cotidal charts for the diurnal and semidiurnal species separately. Figure 29 and 30, respectively, show cotidal charts for the semidiurnal and diurnal tides of the East Coast and Gulf of St. Lawrence. Figures 31 and 32 show the same thing for the West Coast. Large differences are apparent between the semidiurnal and diurnal charts, with respect to location of amphidromes and other characteristics. Clearly, where the tide is of the mixed type (MSD or MD), a cotidal chart that attempted to combine the two species to represent the total tide could cover only a very small region.

As will be discussed more fully in Part II, cotidal charts must often be prepared for use in reduction of soundings to datum on offshore surveys. Since they usually attempt to represent the total tide, the extent of the region they may cover depends upon the extent to which the tide is of the mixed type. In cotidal charts for sounding reduction, times and heights are most often referred to the tide at a nearby reference port. The region is commonly divided into two sets of overlapping zones, one set being zones in which the range is considered to bear a constant ratio to that at the reference port, and the other being zones in which the arrival time of the tide is considered to differ by a constant from that at the reference port.

3.11 Numerical modelling of tides

Numerical modelling is becoming more and more common in modern tidal studies, being encouraged by the ever-increasing capabilities of the electronic digital computer. Models incorporate the physical principles of the equations of motion and of continuity, a description of the shape and the bathymetry of the basin, and a set of boundary conditions that must be preserved. The boundary conditions consist of the harmonic constants at all gauge and current meter sites for which analyses exist, and a statement of the character of the tide along any open boundaries. The region is divided into a set of grid points, closely enough spaced to define the characteristics being investigated. The computer is programmed to commence with an initial set of elevations at all grid points and to change the elevations progressively in accordance with the physical principles and the boundary conditions. In this way, the progression of the tide can be modelled over as many cycles as desired. The tidal streams are also modelled in the same opera-

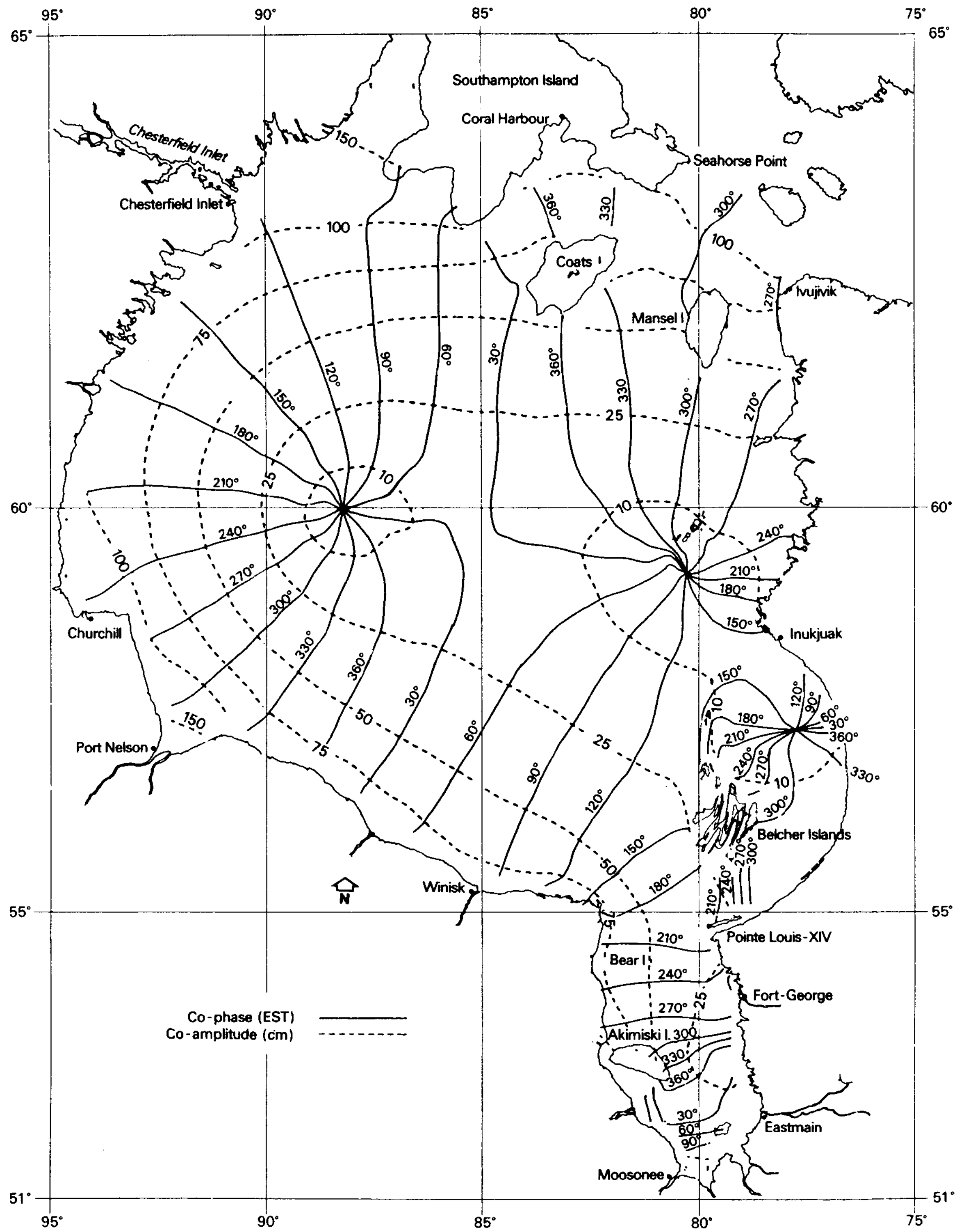

FIG. 27. Cotidal chart of M_2 constituent in Hudson and James Bays by numerical modelling. (from fig. 7 of Freeman and Murty, J. Fish. Res. Board Can. 33(10), 1976).

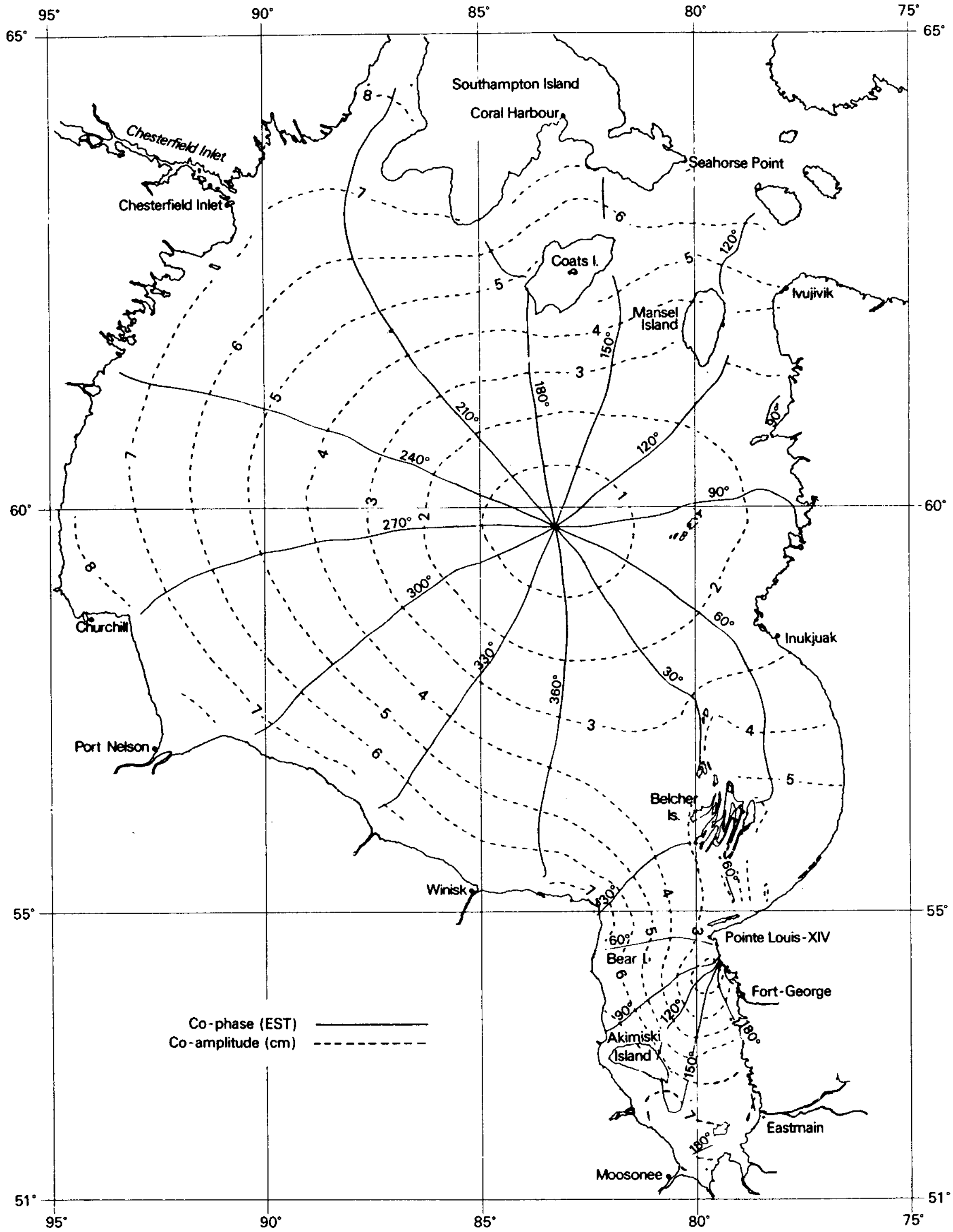

FIG. 28. Cotidal chart of K_1 constituent in Hudson and James Bays by numerical modelling. (from fig. 10 of Freeman and Murty, J. Fish. Res. Board Can. 33(10), 1976).

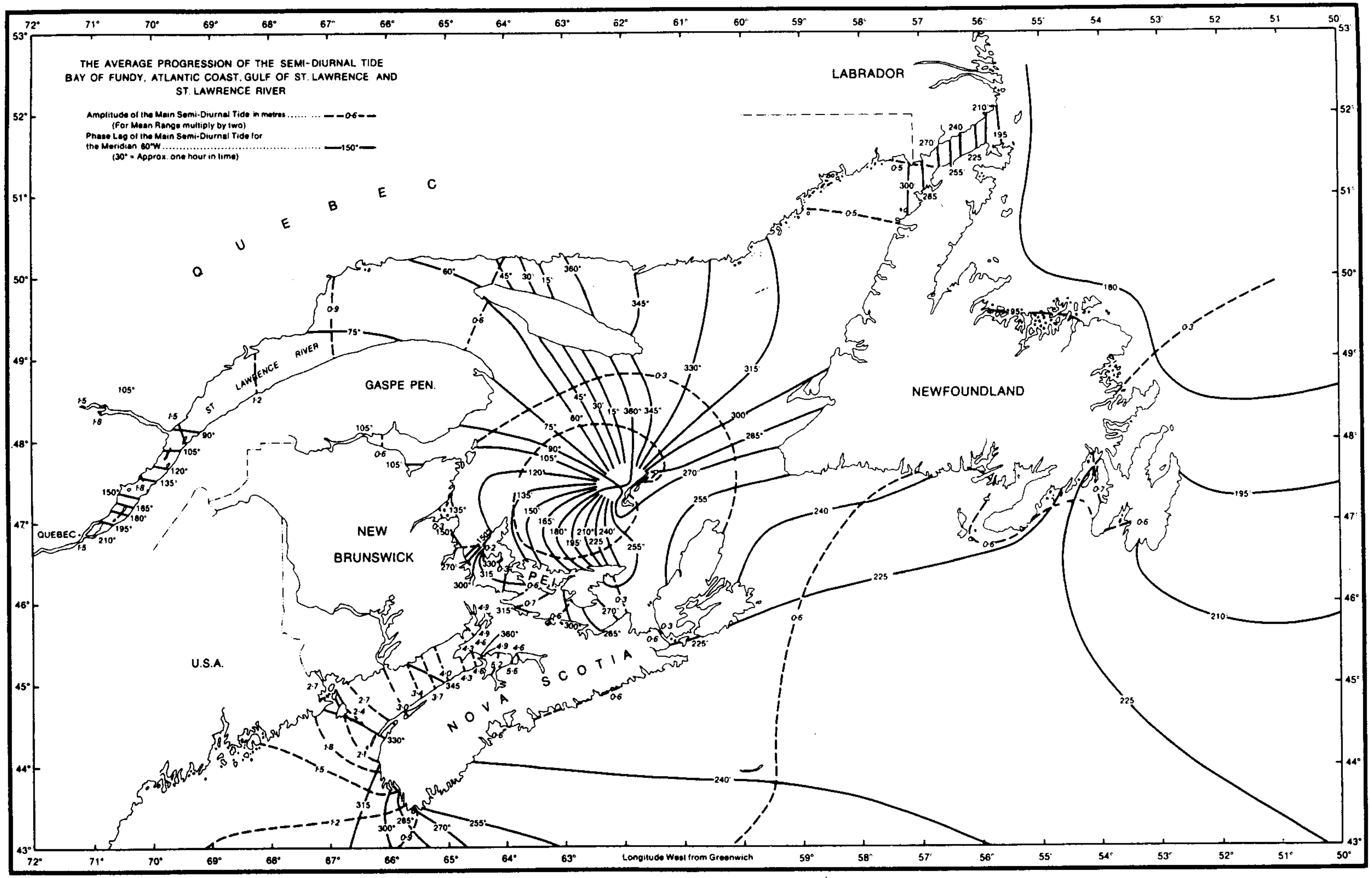

FIG. 29. Cotidal chart of semidiurnal tide on East Coast of Canada. (from fig. 10 of *Tides in Canadian Waters*, by G. Dohler).

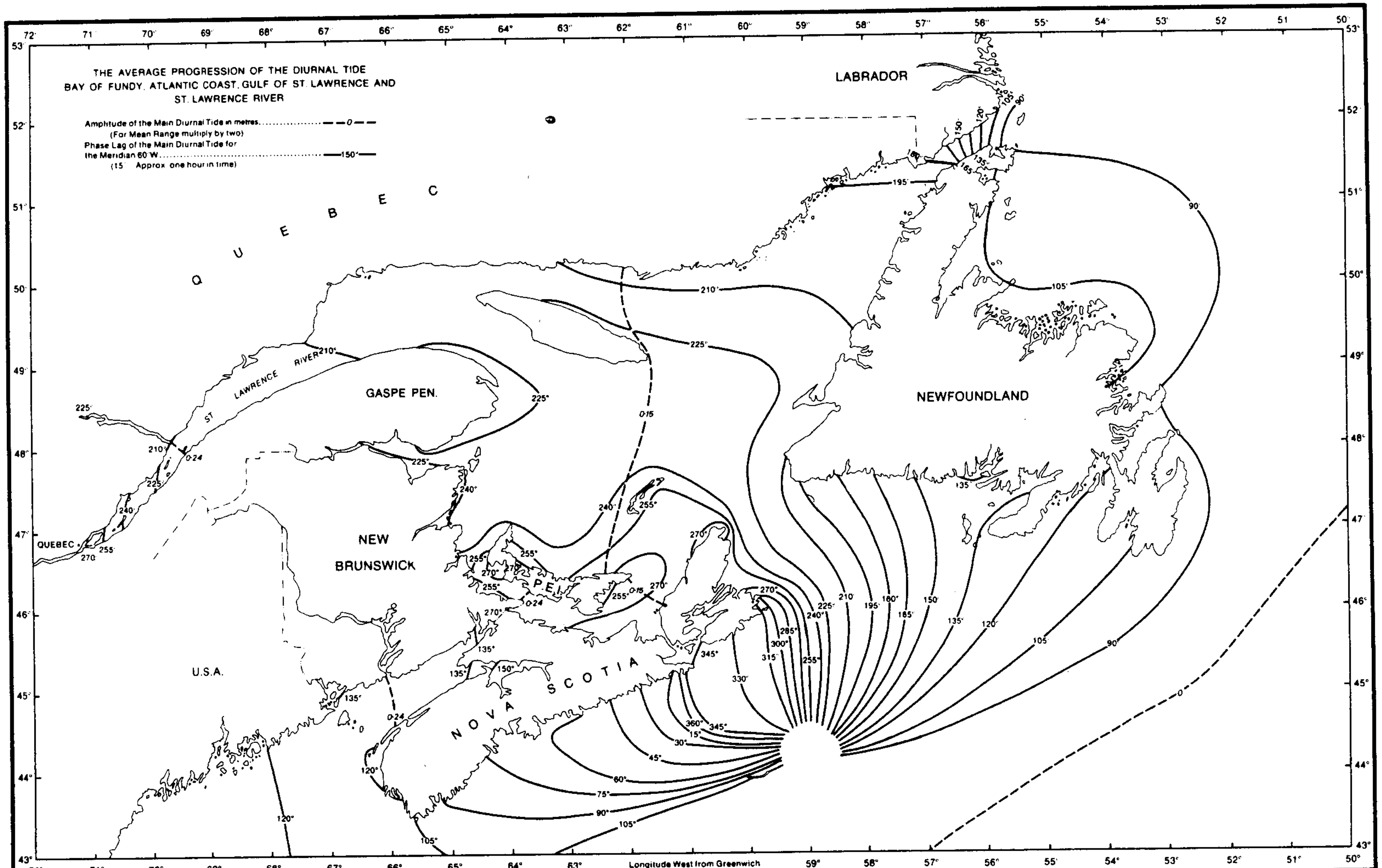

FIG. 30. Cotidal chart of diurnal tide on East Coast of Canada. (from fig. 11 of *Tides in Canadian Waters*, by G. Dohler).

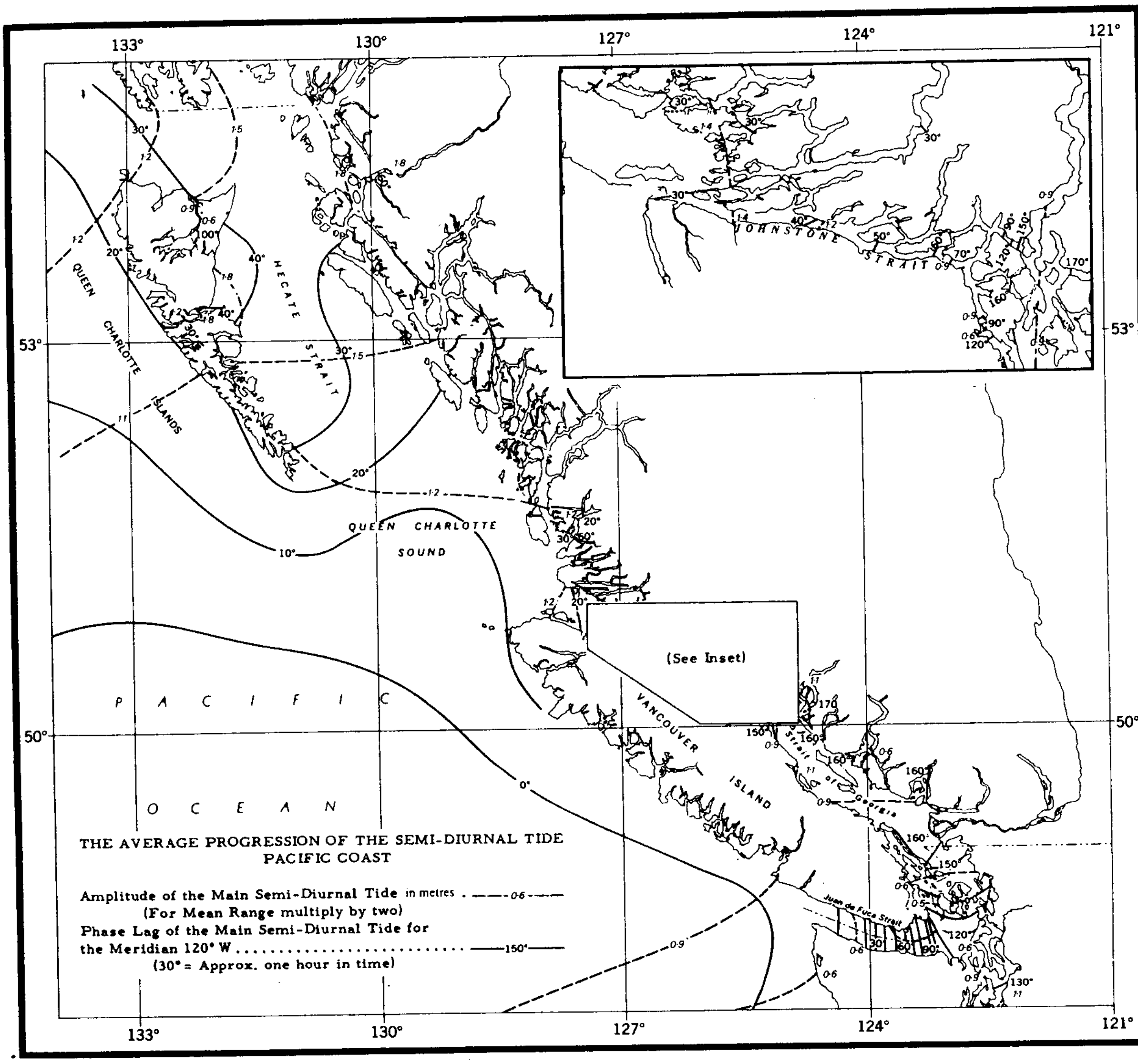

FIG. 31. Cotidal chart of semidiurnal tide on West Coast of Canada. (from fig. 18 of *Tides in Canadian Waters*, by G. Dohler).

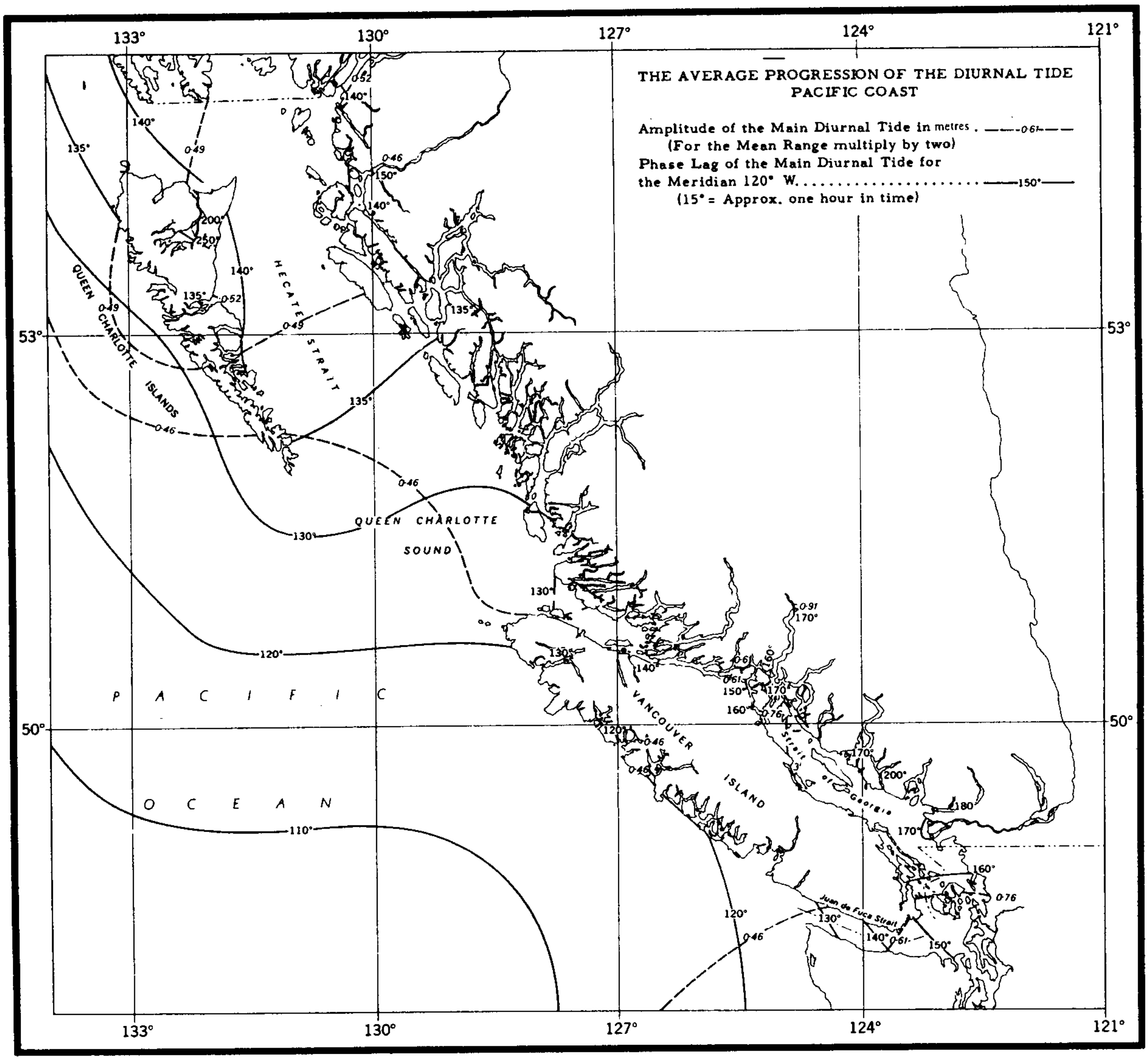

FIG. 32. Cotidal chart of diurnal tide on West Coast of Canada. (from fig. 19 of *Tides in Canadian Waters*, by G. Dohler).

tion, through the application of the principle of continuity. The cotidal charts of Fig. 27 and 28 are the result of numerical modelling of the tides in Hudson Bay. Figure 33 shows current vectors for one stage of the tide in Chignecto Bay and Minas Basin, as deduced by numerical modelling, and Fig. 34 shows a similar result from numerical modelling in the Strait of Georgia and Juan de Fuca Strait.

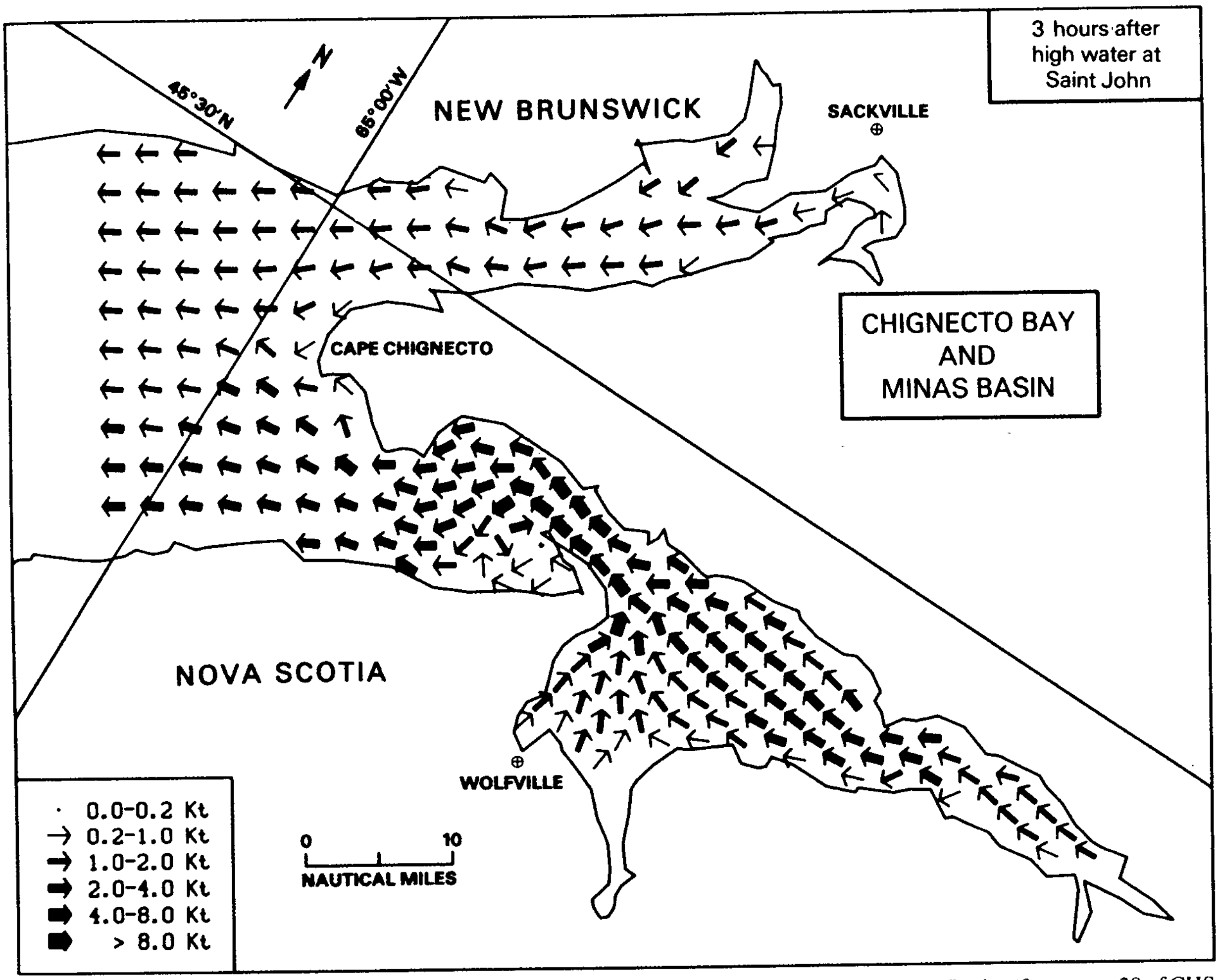

FIG. 33. Sample of tidal current information from numerical modelling in Chignecto Bay and Minas Basin. (from page 28 of CHS *Atlas of Tidal Currents — Bay of Fundy and Gulf of Maine*).

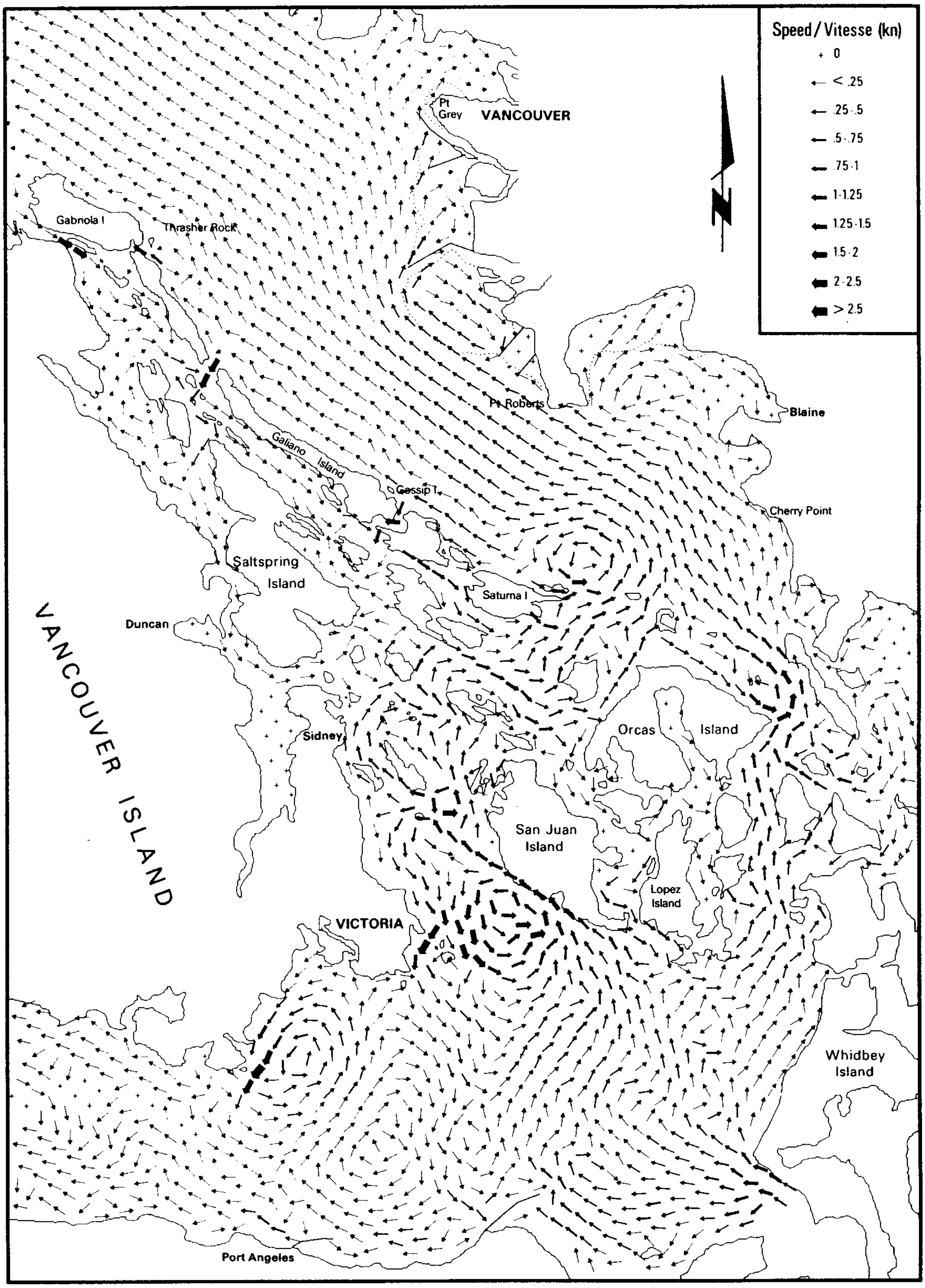

FIG. 34. Sample of tidal current information from numerical modelling in Strait of Georgia and Juan de Fuca Strait. (from page 55 of *CHS Atlas of Currents — Juan de Fuca Strait to Strait of Georgia*, 1983).